International Illiquidity*

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Abstract

We study the role of funding-constrained investors in international financial markets. We proxy the tightness of investors' constraints, or their illiquidity, by the deviations of government bond yields from a fitted yield curve, and construct such measures for a range of countries. In line with the predictions from an international asset pricing model with funding constraints, larger average values of our measures are associated with a lower slope and a higher intercept of the international security market line, while their cross-country variation is associated with cross-country variation in alpha. We also extend the model to study the pricing of international funding illiquidity risk and estimate a significant negative illiquidity risk premium in the cross-section of stock returns. We conclude that both illiquidity level and illiquidity risk matter in international financial markets.

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The recent crisis has shown that the effect of frictions on asset prices is both considerable and pervasive through international financial markets. Researchers and policymakers alike require indicators that measure the severity of these frictions across countries and allow them to uncover and understand the mechanisms at work.

In this paper, we study the international effect of funding frictions. Our analysis is guided by an asset pricing model in which investors face capital constraints. We refer to the tightness of these constraints as illiquidity. Our contribution is threefold. First, we construct daily illiquidity measures for a range of countries that are grounded in our theoretical model. Second, we find that illiquidity distorts the risk–return tradeoff in international stock markets. In line with our model predictions, higher average *level* of illiquidity lowers the slope and increases the intercept of the international security market line, while cross-country variation in the level of illiquidity is associated with cross-country variation in alpha. Third, we study the pricing of global illiquidity *risk*. Theoretically, we show that the premium associated with illiquidity shocks is driven by two offsetting forces. On one hand, investors hedge against their constraints binding in the future. On the other hand, higher illiquidity is associated with higher expected returns and, hence, better investment opportunities, even for constrained investors. Empirically, we find a negative global illiquidity risk premium, indicating that the former effect dominates.

We measure illiquidity by the average squared deviation of observed government bond yields from a fitted yield curve. As argued by Hu, Pan, and Wang (2013), these deviations become large when similar bonds trade at different yields, providing us with a measure of apparent arbitrage opportunities unexploited by investors. In our model, such opportunities emerge in equilibrium because binding funding constraints prevent investors from eliminating price discrepancies between bonds with similar risk. Government bonds are particularly well suited to reveal the severity of funding frictions, because they are actively used by investors both for investment and funding purposes.[1] Moreover, there are multiple government bonds with similar duration outstanding and their yields are normally well described by a simple factor structure, making deviations easy to detect.

We use the same methodology to compute illiquidity measures for the United States, Ger-

[1] The safety and liquidity of government bonds makes them a crucial source of collateral for financing other asset positions. Out of $12 trillion of repo and reverse repo transactions outstanding globally, $9 trillion are collateralised with government bonds, see Committee on the Global Financial System (2017).

many, the United Kingdom, Canada, Japan, and Switzerland at a daily frequency for a history of more than 20 years.[2] We then construct a global illiquidity measure as the market-capitalization weighted average of country-level illiquidity measures. Using these proxies of illiquidity, we find that while country-level illiquidity measures feature a strong common component, they also exhibit variation that is not shared globally. For example, a major increase in the German and UK indicators following the British pound withdrawal from the Exchange Rate Mechanism in September 1992 leaves the US illiquidity measure largely unaffected, as illustrated on Figure 1. This finding suggests that exploiting the cross-country variation in illiquidity can provide us with additional power to test the illiquidity effect on asset prices internationally.

[Insert Figure 1 here.]

Our main hypothesis is that funding frictions proxied by the presence of unexploited apparent arbitrage opportunities also distort the risk-based pricing of international stocks. Moreover, this effect can vary across countries. In our model, investors have to fund a fraction of their position in each asset with their own capital. This capital requirement represents, in reduced form, the combined effect of regulation and market discipline.[3] When this constraint binds for at least some investors, the equilibrium expected excess return on any stock depends not only on its risk (beta), but also on an additional illiquidity component that depends on the capital required to maintain the position in this stock and a multiplier that measures the shadow price of the constraint. We do not assume any additional frictions that could segment financial markets.[4] Even in this case, the level of illiquidity can vary across countries either because of cross-country variation in capital requirements or because it requires more capital to leverage

[2]We focus on these six markets for several reasons. First, these markets represent approximately 84% of the MSCI World capitalisation. Second, we have sufficient data on government bond yields for these countries. Third, because of their safety and liquidity, the government bonds of these countries account for 97% of the $9 trillion global repo transactions backed by government bonds.

[3]Kiyotaki and Moore (1997), Fostel and Geanakoplos (2008), Gârleanu and Pedersen (2011), among many others, highlight the importance of such frictions. In practice, these frictions can take the form of regulatory capital requirements, margins, borrowing limited by agency considerations, etc.

[4]Financial markets became less segmented over the past decades. In particular, Bekaert, Harvey, Lundblad, and Siegel (2011) argue that the group of countries considered in our analysis has been effectively integrated since the early 1990s. This is also supported by the empirical findings in Lee (2011), who documents that global market liquidity risk is more important than local market liquidity risk in developed countries, indicating a high degree of financial integration, or Viceira, Wang, and Zhou (2017), who document high market integration across international stock and bond markets.

cross-border positions compared with domestic positions.[5]

The theory predicts that a higher global illiquidity level is associated with a higher intercept and a lower slope of the international security market line (SML). This effect occurs because capital-constrained investors value securities with higher exposure to the global market factor. Comparing the lowest with the highest quintiles of illiquidity periods, we find that the intercept rises from 0.025% to 0.180% per month, while the slope flattens from 0.025% to -0.150%, with both differences being statistically significant. The prediction is further supported by a formal test where we control for other possible determinants of the SML shape.

Another prediction pertains to the cross-country differences in the level of illiquidity which imply differences in risk-adjusted returns compensating investors for the capital they have to commit to maintain their positions. We find that in the cross-section of illiquidity- and beta-sorted portfolios, holding the beta of a security constant, its alpha increases in country illiquidity. For example, for high-beta stocks, the alpha increases from 0.349% to 0.741% per month from low- to high-illiquidity countries, respectively, with the difference being statistically significant. Similarly, the annualized Sharpe ratio increases from 0.19 to 0.43. Frazzini and Pedersen (2013) argue that the effect of illiquidity can be well summarized by the performance of the so-called betting-against-beta (BAB) portfolios that exploit constrained investors' preference for high-beta assets. Our model predicts that these portfolios perform better in more illiquid countries. We verify that the portfolio implementing the BAB strategy in countries that have high illiquidity in a given period outperforms the portfolio doing the same in low-illiquidity countries. For example, BAB strategies in low (high) illiquidity countries produce an average return of 0.598% (1.368%) per month. Further, a strategy long in the high illiquidity and short in the low illiquidity BAB strategy yields a highly significant monthly return of 0.770% (with t-statistic of 3.00) and a corresponding annualized Sharpe ratio of 0.6.

Next, we study the pricing of illiquidity risk, i.e., illiquidity shocks as opposed to its level. To this effect, we relax the assumption that constrained investors have a myopic investment horizon and derive the determinants of their hedging demand. First, investors hedge against

[5] Fostel, Geanakoplos, and Phelan (2017) analyze the differences in collateral quality across countries, while Akbari, Carrieri, and Malkhozov (2017) consider constraints on investors' ability to access funding for their cross-border positions because such positions imply a higher regulatory capital charge or because foreign collateral is perceived to be of lower quality.

states of the world where the shadow price of an additional dollar of capital is high. Second, they have a standard Merton (1973) intertemporal hedging demand for assets that pay off when investment opportunities become poor. However, as high illiquidity drives expected returns up, the two hedging demands have opposite signs. As a result, the sign and the magnitude of the equilibrium illiquidity risk premium depend on the relative strength of the two hedging motives.[6] Using Fama and MacBeth (1973) regressions, we find that shocks to average global illiquidity command a significant negative risk premium of 43 basis points per month, indicating a strong investor willingness to hedge against the deterioration of global funding conditions.

We employ a series of robustness checks to test the validity of our empirical results. First, to estimate betas, we rely on the method of Vasicek (1973) that shrinks estimated betas towards their mean to minimize measurement error. One might argue that additional prior knowledge is useful to predict future betas. We therefore follow Karolyi (1992), who introduces multiple shrinkage targets based on industry and firm size. Using these betas, we find all our results unaffected or stronger. Second, evidence in Hong and Sraer (2016) shows that investors' disagreement, combined with the presence of short sale constraints, decreases the slope of the SML. Controlling for the disagreement proxy proposed by the authors, we find that the significance of global illiquidity for the intercept and the slope of the SML is unaffected. Third, since funding conditions could be correlated with market illiquidity, we control for the effect of the latter by orthogonalizing our illiquidity indicators with respect to the Amihud (2002) stock market illiquidity measure. Using the orthogonalized indicators, we find our theoretical predictions still confirmed in the data.[7] Fourth, it is well known that omitted variables can bias the estimated price of risk. The literature suggests two remedies: adding factors to cross-sectional regressions

[6] The intuition for why investors may not be willing to pay a premium to hedge against aggregate illiquidity risk is similar to Begenau (2016), who shows that higher capital requirements can reduce overall bank funding costs by constraining the supply of safe liquid assets by the banks to be closer to the monopolistic outcome. In the author's model, higher requirements benefit the banking sector as a whole, although each bank would prefer its own requirements relaxed.

[7] The possible link between funding and market liquidity is also discussed in Karolyi, Lee, and van Dijk (2012) who study commonality in market liquidity across countries and find little evidence that commonality is greater in times of higher local interest rates, which represent tighter credit conditions when financial intermediaries are more likely to hit their capital constraints. Moreover, commonality is also not related to changes in the financial health of funding agents like local banks or global prime brokers. In the fixed income market, Dudley (2016) notes that measures of bond market illiquidity such as bid-ask spreads are in general very stable and only showed significant correlation with proxies of funding liquidity (volume of dealer-funded repos) during the 2008/2009 crisis. Using European government bond market data, Moinas, Nguyen, and Valente (2017) find that funding illiquidity shocks may affect market illiquidity but that the reverse feedback in general is weak.

known to affect expected returns or adjusting the estimates. We apply both methods and find that neither changes the significance of global illiquidity as a priced risk factor; the estimated price of global illiquidity risk is negative and highly statistically significant.

The rest of the paper is organized as follows. After a literature review, Section 1 describes the model and derives its predictions. Section 2 describes the data and the construction of the illiquidity proxies. Section 3 analyzes the illiquidity measures, while Section 4 presents our empirical results. Finally, Section 5 concludes. All proofs are deferred to the Appendix. Additional results are available in an Online Appendix.

Related literature: Our work is related to several literature strands. Adrian and Shin (2010), Pasquariello (2014), and Miranda-Agrippino and Rey (2015), among others, propose a range of funding liquidity and market stress measures. Unlike measures that suffer from short time series (e.g., implied volatility indices such as the VIX), are only available at very low frequency (e.g., broker-dealers' leverage), are difficult to compare internationally (e.g., TED spread), or are influenced by particular events (e.g., CIP deviations, which were not present before the most recent financial crisis and later were significantly reduced by central bank foreign exchnage swap lines), our measures are constructed using the same methodology in all countries and are available daily for a history of more than 20 years.

Our paper speaks to the theoretical literature that links funding frictions to asset returns. For instance, Gârleanu and Pedersen (2011) and Frazzini and Pedersen (2013) show that funding constraints can lead to violations of the Law of One Price and deviations from risk-based pricing, respectively. We apply the theoretical insights of these papers to an international setting. Moreover, we relax the assumption that constrained investors have a myopic investment horizon and study the pricing of funding illiquidity risk. To the best of our knowledge, funding illiquidity risk pricing has not been comprehensively addressed in the theoretical literature. Brunnermeier and Pedersen (2009) show that a constrained risk neutral investor values assets whose returns covary positively with the future shadow prices of her constraint above their expected payoffs. Malamud and Vilkov (2017) derive the Merton (1973) hedging demand that arises because illiquidity affects investment opportunities. A similar hedging demand term is also a key element in Kondor and Vayanos (2015). We show that the equilibrium illiquidity risk premium depends on both effects described above. Moreover, these two effects tend to offset each other.

Our focus on funding illiquidity is complementary to a vast literature on market illiquidity. Pástor and Stambaugh (2003) and Acharya and Pedersen (2005) find that aggregate market liquidity is a priced risk factor in the U.S. stock market and carries a positive premium, although the two studies disagree on the economic magnitude of this premium. Sadka (2010) and Franzoni, Nowak, and Phalippou (2012) find similar results for hedge-fund and private-equity returns, respectively. For more references, see Vayanos and Wang (2013) who survey the theoretical and empirical literature on market liquidity. One corollary implication of our analysis is that the theory of funding illiquidity risk pricing is different from that of market illiquidity risk as outlined, for instance, in Acharya and Pedersen (2005). Empirically, we find a negative and significant funding illiquidity risk premium comparable in magnitude to Pástor and Stambaugh (2003). Bekaert, Harvey, and Lundblad (2007), Lee (2011), Goyenko and Sarkissian (2014), and Amihud, Hameed, Kang, and Zhang (2015), study market illiquidity in an international context. We show that funding illiquidity has an important effect on international stock returns, even after controlling for the effect of market illiquidity.

1 Model

1.1 International CAPM with Leverage Constraints

We index time by t, investors by i, countries by j, stocks by k, and bonds by h. To simplify the notation, we assume the information about the corresponding country is already contained in indices k and h, and hence only emphasize country index j when needed.

Assets. Time is discrete, goes from zero to infinity, and is indexed by $t = 0, 1, ...$ We consider a world economy with a set of countries $\mathcal{J}$. In each country $j \in \mathcal{J}$ there exist a set of stocks $k \in \mathcal{K}_j$. We denote the set of all stocks by $\mathcal{K} \equiv \cup_{j \in \mathcal{J}} \mathcal{K}_j$. At date t, stock $k \in \mathcal{K}$ is in supply $\theta_t^k > 0$, it pays a random real dividend D_t^k in the unique consumption good, and its ex-dividend price is P_t^k. Agents also have access to a riskless asset with the instantaneous net return (i.e., short rate) process r_t given exogenously. Finally, we assume that purchasing power parity holds and all prices are expressed in US dollars.[8]

[8]See, e.g., Bekaert, Harvey, and Lundblad (2007), who make a similar assumption in the context of interna-

Agents. Stocks are held by overlapping generations of international investors. Each generation lives for two periods; in period t, agents invest to maximize mean-variance preferences over next period wealth, then consume and exit in period $t+1$. Investor $i \in \mathcal{I}$, born with wealth $W_{i,t} \geq 0$, can invest in all assets of the world economy. If $x_{i,t}^k$ denotes the dollar amount investor i holds in stock k at time t, she maximizes

$$\max_{\left\{x_{i,t}^k\right\}_{k\in\mathcal{K}}} \mathrm{E}_t\left[W_{i,t+1}\right] - \frac{\alpha}{2}\mathrm{Var}_t\left[W_{i,t+1}\right] \tag{1}$$

subject to her budget constraint

$$W_{i,t+1} = W_{i,t}\left(1+r_t\right) + \sum_{k\in\mathcal{K}} x_{i,t}^k\left(R_{t+1}^k - r_t\right), \tag{2}$$

where $R_{t+1}^k \equiv \left(D_{t+1}^k + P_{t+1}^k\right)/P_t^k - 1$ denotes the net return on stock k. Furthermore, investing in or shorting securities requires investor i to commit the amount of capital equal to the multiple $m_{i,t}^k$ of the position size:

$$\sum_{k\in\mathcal{K}} m_{i,t}^k\left|x_{i,t}^k\right| \leq W_{i,t}. \tag{3}$$

Investor i's capital requirements differ across countries but are the same inside each country: $m_{i,t}^k = m_{i,t}^j$ for $k \in \mathcal{K}_j$. In absence of explicit investment barriers or currency risk, capital requirements are the only dimension along which countries differ from investors' perspective.

Equilibrium. We write stock return dynamics in the form

$$R_{t+1}^k = \mu_t^k + \left(\sigma_t^k\right)^\top \varepsilon_{t+1}, \tag{4}$$

where $\mu_t^k \in \mathbb{R}$ is the expected net return of stock k, ε_{t+1} is an N-dimensional random vector with mean zero and the identity covariance matrix that collects all relevant uncertainty of the economy, $\sigma_t^k \in \mathbb{R}^N$ is an N-dimensional vector, $\top$ denotes transpose, and thus $\mathrm{Cov}_t\left[R_{t+1}^k, R_{t+1}^{k'}\right] = \left(\sigma_t^k\right)^\top \sigma_t^{k'}$ is the covariance between the returns of two stocks k and k'. Substituting (4) into (2) and then into (1), and denoting the Lagrange multiplier of (3) by $\psi_{i,t}$, we obtain the following lemma:

tional asset pricing.

Lemma 1. *The first-order conditions of agent i imply*

$$\mu_t^k - r_t = \alpha \left(\sigma_t^k\right)^\top \sum_{k' \in \mathcal{K}} x_{i,t}^{k'} \sigma_t^{k'} + \psi_{i,t} m_{i,t}^k sgn\left(x_{i,t}^k\right) \tag{5}$$

where $sgn(x) = \pm 1$ if $x \gtrless 0$ and $sgn(x) \in [-1, 1]$ if $x = 0$.

Equation (5) states that the expected excess return investors require when investing in stock k, $\mu_t^k - r_t$, consists of two terms. The first term is a compensation for the risk of stock k: it is proportional to the return volatility, σ_t^k, and the proportionality term depends on the aggregate amount of risk investor i takes, captured by $\sum_{k' \in \mathcal{K}} x_{i,t}^{k'} \sigma_t^{k'}$. The second term depends on how constrained investor i is: it is zero if the constraint does not bind ($\psi_{i,t} = 0$); otherwise it depends on whether investor i is long or short stock k, captured by $\operatorname{sgn}\left(x_{i,t}^k\right)$, and is larger in absolute terms for assets with higher capital requirement $m_{i,t}^k$. Intuitively, being long one dollar in a stock with a higher requirement ties down more capital of the investor, who then requires a higher compensation on this asset.

The market-clearing conditions for all t and k are given by

$$\sum_{i \in \mathcal{I}} x_{i,t}^k = \theta_t^k. \tag{6}$$

Aggregating (5) across all investors $i \in \mathcal{I}$ and imposing market-clearing condition (6), we get

$$\mu_t^k - r_t = \frac{\alpha}{|\mathcal{I}|} \left(\sigma_t^k\right)^\top \sum_{k' \in \mathcal{K}} \theta_t^{k'} \sigma_t^{k'} + \frac{1}{|\mathcal{I}|} \sum_{i \in \mathcal{I}} \psi_{i,t} m_{i,t}^k \operatorname{sgn}\left(x_{i,t}^k\right), \tag{7}$$

where $|\mathcal{I}|$ denotes the number of investors. Let us define a global market index, $G_t \equiv \sum_{k' \in \mathcal{K}} \theta_t^{k'}$, that is the dollar-supply weighted average of all stocks, and denote the return on this index by

$$R_{t+1}^G = \sum_{k' \in \mathcal{K}} \frac{\theta_t^{k'}}{G_t} R_{t+1}^{k'}. \tag{8}$$

Combining (7) and (8), after some algebra, we obtain:

Theorem 1. *The equilibrium expected excess return of security k is*

$$\mu_t^k - r_t = \beta_t^k \lambda_t + \phi_t^k - \beta_t^k \phi_t^G, \tag{9}$$

with

$$\phi_t^k = \frac{1}{|\mathcal{I}|} \sum_{i \in \mathcal{I}} \psi_{i,t} m_{i,t}^k sgn\left(x_{i,t}^k\right) \;\; and \;\; \phi_t^G = \sum_{k' \in \mathcal{K}} \frac{\theta_t^{k'}}{G_t} \phi_t^{k'}, \tag{10}$$

where $\beta_t^k = Cov_t\left[R_{t+1}^k, R_{t+1}^G\right] / Var_t\left[R_{t+1}^G\right]$ *is the conditional beta of security* k *with respect to the global market portfolio, and* $\lambda_t = \mu_t^G - r_t$ *is the global market portfolio expected excess return.*

Predictions. In the following, we derive some empirical predictions from Theorem 1 that we can test in the data. To this end, we assume that $x_{i,t}^k > 0$, in line with the evidence in Frazzini and Pedersen (2013). We refer to the terms that capture this effect, $\phi_t^k = \phi_t^j$ for $k \in \mathcal{K}_j$, as country illiquidity, and its average, ϕ_t^G, as global illiquidity.

Remark 1. *Illiquidity can differ across countries even in integrated global markets.*

Because there are no explicit barriers that segment international stock markets, the effect of leverage constraints on the expected return of security k in country j is the average of all investors' Lagrange multipliers $\psi_{i,t}$, weighted by their respective capital requirements for this country $m_{i,t}^j$, as can be seen from (10). Differences in capital requirements $m_{i,t}^j \neq m_{i,t}^{j'}$ lead to differences in country illiquidity $\phi_{i,t}^j \neq \phi_{i,t}^{j'}$. Differences in the shadow price of leverage $\psi_{i,t}$ across investors, however, do not necessarily lead to differences in illiquidity. To see this, consider the case where for all investors and countries capital requirements can be written as $m_{i,t}^j = m_{i,t} m_t^j$. In this case the same combination of Lagrange multipliers matters for illiquidity in all countries: $\phi_t^j = m_t^j \sum_{i \in \mathcal{I}} \psi_{i,t} m_{i,t} / |\mathcal{I}|$. However, if this condition is not satisfied, heterogeneity in the tightness of leverage constraints across investors contributes to differences in illiquidity across countries. In particular, this occurs when capital requirements are higher for cross-border positions compared to domestic positions: $m_{i',t}^j > m_{i,t}^j$ for investor i located in country j, and investor i' located abroad.

Proposition 1. *There is an average global security market line (SML) with slope decreasing in global illiquidity and intercept increasing in global illiquidity.*

Proposition 1 is a rearrangement of (9) in Theorem 1:

$$\mu_t^k - r_t = \phi_t^G + \beta_t^k \left(\lambda_t - \phi_t^G\right) + \left(\phi_t^k - \phi_t^G\right).$$

As the dollar-supply weighted average of the $\phi_t^k - \phi_t^G$ terms is zero by construction, the intercept of the SML is given by ϕ_t^G, and the slope is $\lambda_t - \phi_t^G$. Equation (9) also yields:

Proposition 2. *Holding illiquidity constant, a higher beta means lower alpha. Because securities can lie off the security market line due to the country-specific term ϕ_t^j, holding beta constant, the alpha increases in the asset illiquidity.*

Alphas with respect to the global market, $\phi_t^k - \beta_t^k \phi_t^G$, arise because constrained investors pay a premium for high-beta stocks that allow them to get a higher exposure to the global market factor per unit capital. For the same reason, investors require additional compensation for securities in high-illiquidity countries. The combination of these two effects characterize the distribution of risk-adjusted returns across securities.

Finally, we derive a proposition regarding the performance of portfolios constructed to take advantage of these risk-adjusted returns.[9]

Proposition 3. *Everything else being equal, the expected return of a self-financing market-neutral portfolio that is long in low-beta securities and short in high-beta securities in country j, with the appropriate leverage applied to the two legs, is positive and increasing in country illiquidity.*

Proposition 3 states that the BAB portfolio proposed by Frazzini and Pedersen (2013) has higher expected returns in countries where investing is more difficult to fund. To see this, consider two portfolios composed of country-j securities with average betas $\beta_t^{HB} > \beta_t^{LB}$. The expected return on the BAB portfolio is then:

$$\mu_t^{j,BAB} = \frac{1}{\beta_t^{LB}} \left(\beta_t^{LB} \lambda_t + \phi_t^j - \beta_t^{LB} \phi_t^G \right) - \frac{1}{\beta_t^{HB}} \left(\beta_t^{HB} \lambda_t + \phi_t^j - \beta_t^{HB} \phi_t^G \right) = \phi_t^j \frac{\beta_t^{HB} - \beta_t^{LB}}{\beta_t^{LB} \beta_t^{HB}} > 0.$$

1.2 Model-implied Measure of Illiquidity

To motivate the use of yield curve "noise" as a measure of illiquidity introduced in Section 2, this section augments the model with a set of zero-coupon bonds $h \in \mathcal{H}_j$ in each country j.

[9]These strategies represent a genuine trading opportunity only for unconstrained investors, who do not require a compensation for the shadow cost of the capital constraint.

We denote the time-t yield-to-maturity of the zero-coupon bond h of country j, that pays one dollar at maturity $t+\tau_h$, by $y_t^{j,h}$.

Constrained investors, with objective (1), can hold both stocks and bonds. Investing in or shorting bond h of country j requires investor i to commit the amount of her capital equal to the multiple $m_{i,t}^{j,h}$ of the position size $\left|z_{i,t}^{j,h}\right|$. In equilibrium, agents need to hold the supply of bond h net of any buy-and-hold or preferred-habitat agent holdings, denoted by $d_t^{j,h}$, which can be positive or negative.[10]

We also make additional tractability assumptions that allow us to derive a simple closed-form expression for the yield curve. The dynamics of the short rate r_t under the physical probability measure are given by the mean-reverting process

$$r_{t+1} = r_t + \kappa\left(\bar{r} - r_t\right) + \sigma\eta_{t+1}, \tag{11}$$

where κ, $\bar{r}$, and σ are positive constants, and η_{t+1} and ε_{t+1} are independent. Moreover, for simplicity, we assume $m_{i,t}^k$, θ_t^k, $m_{i,t}^{j,h}$, $d_t^{j,h}$, and $W_{i,t}$ to be constant over time (we defer the discussion of illiquidity dynamics to the next section).[11] Under these assumptions, we obtain the following result:

Theorem 2. *There exists an equilibrium in which the yield on bond h of country j is given by*

$$y_t^{j,h} = \mathcal{A}\left(\tau_h\right) + \mathcal{B}\left(\tau_h\right) r_t + \mathcal{C}_{j,h}\left(\tau_h\right),$$

where the first two terms describe a standard affine yield curve that depends only on the maturity of a bond and the short-rate factor r_t, and

$$\mathcal{C}_{j,h}\left(\tau_h\right) = \frac{1}{\tau_h\left|\mathcal{I}\right|} \sum_{\substack{h' \in \mathcal{H}_j: \\ \tau_{h'} \leq \tau_h}} \sum_{i \in \mathcal{I}} \psi_i m_i^{j,h'} sgn\left(z_i^{j,h'}\right)$$

represents a deviation from this smooth yield curve, specific to country j and bond h.

[10]For instance, to satisfy other investors' demand, primary dealers in the US Treasuries, who are likely to be the marginal investor in this market, hold long positions in some maturities and short positions in others, as can be seen from the FR 2004 reports.

[11]We can allow η_t and ε_t to be correlated, r_t to be driven by multiple factors, and the net supply of bonds d_t^h to be upward-sloping. However, this only introduces additional technical complexity without adding to the economic intuition or altering our main message.

The $\mathcal{C}_{j,h}(\tau_h)$ term in Theorem 2 arises because, by analogy with Lemma 1 for stocks, each investor i is compensated not only for bond risk, but also for the capital $m_i^{j,h}$ she must commit, with higher (lower) yields required for long (short) positions. Thus, the average magnitude of deviations $\mathcal{C}_{j,h}(\tau_h)$ across all bonds in country j is higher when its capital requirements and investors' shadow price of leverage are high, connecting "noise" in the term structure to the CAPM alphas of stocks.

1.3 Non-Myopic Investors

Since our theory predicts that funding illiquidity level has an effect of expected returns, it is natural to explore if it has any predictions regarding the pricing of funding illiquidity risk. To this end, we consider a modified version of the baseline Section 1.1 model in which we relax the assumption that investors have a myopic investment horizon and allow illiquidity to vary over time both through endogenous changes in investors' wealth or due to an exogenous tightening of funding constraints. More specifically, instead of a sequence of static mean-variance tradeoffs, we assume that all investors live for three periods, $t = 0, 1, 2$, and trade at dates $t = 0, 1$ to maximize utility over date-2 wealth:[12]

$$\max_{\left\{x_{i,t}^k\right\}_{k\in\mathcal{K}}} \mathrm{E}\left[W_{i,2}\right] - \frac{\alpha}{2}\mathrm{Var}\left[W_{i,2}\right], \tag{12}$$

subject to the budget and funding constraints

$$W_{i,t+1} = W_{i,t} + \sum_{k\in\mathcal{K}} x_{i,t}^k R_{t+1}^k \tag{13}$$

and

$$\sum_{k\in\mathcal{K}} m_{i,t}^k \left|x_{i,t}^k\right| \leq W_{i,t} + \zeta_{i,t}, \tag{14}$$

where $\zeta_{i,t}$ are zero-mean shocks, uncorrelated with other shocks in the model, that exogenously tighten or relax investor i's constraint, and for expositional reasons we normalize the riskfree rates to $r_t = 0$.

[12] We consider a 3-period model instead of an OLG economy for tractability reasons, to avoid market clearing across two constrained generations actively trading at any time. Our model can easily be rewritten that way, but the additional technical complexity does not yield any new economic insights.

At $t = 1$, agent i's objective is the same as the objective of the myopic investor, and her value function is given by

$$V_{i,1} = \sup_{\left\{x_{i,1}^k\right\}_{k\in\mathcal{K}}} \mathrm{E}_1\left[W_{i,2}\right] - \frac{\alpha}{2}\mathrm{Var}_1\left[W_{i,2}\right]. \tag{15}$$

As a result, date-1 optimal portfolios and equilibrium expected returns are analogous to the one implied by (5) and (7) of the myopic model. In contrast, at $t = 0$, investor i maximizes the time-consistent objective defined following Basak and Chabakauri (2010):

$$\max_{\left\{x_{i,0}^k\right\}_{k\in\mathcal{K}}} \mathrm{E}_0\left[V_{i,1}\right] - \frac{\alpha}{2}\mathrm{Var}_0\left[\mathrm{E}_1\left[W_{i,2}\right]\right] \tag{16}$$

subject to (13) and (14).

The impact of the asset-k holding $x_{i,0}^k$ on the constrained objective is threefold. First, by the envelope theorem, $\partial V_{i,1}/\partial W_{i,1} = 1 + \psi_{i,1}$, which, together with the budget constraint, leads to

$$\frac{\partial \mathrm{E}_0\left[V_{i,1}\right]}{\partial x_{i,0}^k} = \mathrm{E}_0\left[\left(1+\psi_{i,1}\right)R_1^k\right] = \left(1+\mathrm{E}_0\left[\psi_{i,1}\right]\right)\mu_0^k + \ \mathrm{Cov}_0\left[R_1^k,\psi_{i,1}\right].$$

Second, using (13), we also write

$$\frac{1}{2}\frac{\partial \mathrm{Var}_0\left[\mathrm{E}_1\left[W_{i,2}\right]\right]}{\partial x_{i,0}^k} = \mathrm{Cov}_0\left[\frac{\partial \mathrm{E}_1\left[W_{i,2}\right]}{\partial x_{i,0}}, \mathrm{E}_1\left[W_{i,2}\right]\right] = \mathrm{Cov}_0\left[\left(1+\xi_{i,1}\right)R_1^k, W_{i,1} + \sum_{k'\in\mathcal{K}} x_{i,1}^{k'}\mu_1^{k'}\right],$$

where $\xi_{i,1} = \partial\left(\sum_{k\in\mathcal{K}} x_{i,1}^k\mu_1^k\right)/\partial W_{i,1}$ is the impact of an additional dollar on the optimal risky portfolio at date 1. Third, a Lagrange multiplier term captures how constrained investor i is at date 0, just like in the myopic case. Overall, the optimization of this objective gives rise to inter-temporal hedging motives:

Lemma 2. *Investor i's first order condition with respect to her date-0 optimal stock holding $x_{i,0}^k$ implies*

$$\left(1+E_0\left[\psi_{i,1}\right]\right)\mu_0^k = -Cov_0\left[R_1^k,\psi_{i,1}\right] + \alpha\, Cov_0\left[\left(1+\xi_{i,1}\right)R_1^k, E_1\left[W_{i,2}\right]\right] + \psi_{i,0} m_{i,0}^k sgn\left(x_{i,0}^k\right), \tag{17}$$

where $\psi_{i,0}$ denotes the $t = 0$ Lagrange multiplier of (14).

Lemma 2 shows that the date-0 required return on any asset is driven by a non-myopic investor's hedging demand that has two components. First, her valuation of an additional dollar at $t = 1$ depends on the shadow price of her leverage constraint. As a result, everything else being equal, she accepts lower returns on assets that co-vary positively with her multiplier. Second, investor i has the Merton (1973) hedging demand and requires higher returns on assets that co-vary positively with future expected returns on her portfolio.[13] Importantly, equilibrium expected returns at $t = 1$ themselves depend on the tightness of the leverage constraints, as can be seen from (7). Thus, the two hedging motives work to offset each other: On one hand, investors value additional pay-off in the states of the world where their own constraint binds. On the other hand, investment opportunities improve, even for constrained investors, when tighter constraints across investors drive expected returns up. This intuition can be illustrated with the following result:[14]

Theorem 3. *Assuming $m_{i,t}^k = m^k$ for all i and t, and $x_{i,t}^k > 0$ for all i, k and t, equilibrium expected excess returns at date 0 are given by*

$$\mu_0^k = \chi_G Cov_0\left[R_1^k, R_1^G\right] + \chi_\Psi Cov_0\left[R_1^k, \Psi_1\right] + \chi_0^k, \tag{18}$$

where

$$\Psi_1 = \frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}}\psi_{i,1}, \quad \chi_G = \frac{1}{1+E_0\left[\Psi_1\right]}\frac{\alpha}{|\mathcal{I}|}G_0, \quad \chi_\Psi = \frac{1}{1+E_0\left[\Psi_1\right]}\left(\frac{\alpha}{|\mathcal{I}|}\sum_{k'\in\mathcal{K}}\theta_1^{k'}m^{k'} - 1\right), \tag{19}$$

and χ_0^k is given in the Appendix.

The proof detailed in the Appendix consists of aggregating the first order conditions from Lemma 2 across all investors i, and imposing market clearing. Theorem 3 implies that global illiquidity, measured by Ψ_1, is a priced risk factor. The sign of the illiquidity risk premium χ_Ψ reveals the relative strength of the two effects: investors' Merton (1973) hedging demand

[13] With mean-variance preferences, the sign of this hedging demand is the same for all values of the risk aversion coefficient.

[14] To derive equilibrium in closed form, we assume constant capital requirements and long investors' positions in equilibrium, but we allow funding liquidity to vary over time either through endogenous variation in investors' wealth or the exogenous shocks $\zeta_{i,1}$. Nonetheless, the result is more general, as it follows from Lemma 2 that does not rely on additional tractability assumptions.

and their willingness to hedge against the deterioration of global funding conditions. When the Merton term dominates, we have $\chi_\Psi > 0$. When the second effect dominates, $\chi_\Psi < 0$.

2 Data

Bond and Stock Data. We collect data on government bond prices and stock returns from Datastream. The data spans six different countries: the United States, Germany, the United Kingdom, Canada, Japan, and Switzerland. The frequency is daily, running from 1 January 1990 to 31 December 2013, leaving us with 6,048 observations in the time-series.

The country choice is driven by two main factors: data availability and credit risk considerations. For example, while there is enough data available on some Eurozone countries, these sovereign bonds feature quite a large credit risk component, especially after 2008 (see, e.g., Pelizzon, Subrahmanyam, Tomio, and Uno (2016)), an aspect absent from our model. Moreover, according to Bank for International Settlements (2017), there are currently around \$12 trillion of repo and reverse repo transactions outstanding globally, of which nearly \$9 trillion are collateralized with government bonds. The six countries we study comprise of 97% of the total share in repo transactions.

We obtain a daily cross-section of end-of-day bond prices for our sample period for all available maturities. We use mid prices to avoid any discrepancies between the prices of similar bonds due to the bid-ask spread. Following Gürkaynak, Sack, and Wright (2007), we apply data filters, described in Appendix A, in order to account for institutional differences across countries and obtain securities with similar characteristics. Panel A of Table 1 provides details of our international bond sample. We note that on average we have 71 bonds for each country and each day to fit the yield curve, and 60 bonds to construct the illiquidity measure.

[Insert Table 1 here.]

For stocks, we collect daily returns, volume, and market capitalization data for the six countries. The initial sample covers more than 10,000 stocks to which we apply a set of standard filters, described in Appendix A. Excess returns are calculated in excess of the US Treasury bill rate and the proxy for the global market is the MSCI world index. Panel B of Table 1 reports summary statistics. We follow Frazzini and Pedersen (2013) to estimate stock betas; see Appendix A. Alternative methods to estimate betas are discussed in Section 4.4.

Illiquidity Measures. In line with our model, we measure illiquidity as deviations from a smoothed yield curve. To this end, we construct country illiquidity measures following Hu, Pan, and Wang (2013), who employ the Svensson (1994) method, described in detail in Appendix A, to fit the term structure of interest rates. It is well-known from U.S. data, that the cross-sectional and time-series of bond yields are well-described by three factors, usually referred to as level, slope, and curvature (see, e.g., Litterman and Scheinkman (1991)). Since the Svensson (1994) method assumes that yields are driven by three factors, we check in our sample of international government bonds, whether three factors are sufficient by means of principal component analysis. We find that the first principal component (PC) explains almost the majority of bond yield levels in each country. The fraction explained ranges from 95% in Canada and Germany to 98% in Japan. The first two PCs explain more than 99% in each of the countries. A similar picture emerges for the bond yield changes where the first three PCs explain between 85% (Japan) to 95% (Canada, UK and US) of the variation. We conclude that a three factor model à la Svensson (1994) is an appropriate choice to fit yields for the six countries.

In line with our theoretical motivation in Theorem 2, we proxy funding illiquidity as deviations from a smooth yield curve. To this end, we estimate the set of parameters b_t^j of the Svensson (1994) model for each country j and day t, and we minimize the weighted sum of the squared deviations between actual and model-implied prices:

$$b_t^j = \arg\min_b \sum_{h=1}^{H_t^j} \left[\left(P^h(b) - P_t^h \right) \times \frac{1}{D_t^h} \right]^2,$$

where H_t^j denotes the number of bonds available in country j on day t, $P^h(b)$ is the model-implied price for bond $h = 1, ..., H_t^j$, P_t^h is its observed bond price, and D_t^h is the corresponding Macaulay duration. We verify that our yield curve estimates are reasonable by comparing our term structures with the estimates published by central banks or the international yield curves used in Wright (2011) and Pegoraro, Siegel, and Tiozzo 'Pezzoli' (2013).[15]

The illiquidity measure for country j is then defined as the root mean square error between the model-implied yields and the market yields, i.e.,

$$\text{Illiq}_t^j = \sqrt{\frac{1}{H_t^j}\sum_{h=1}^{H_t^j}\left[y^h\left(b_t^j\right) - y_t^h\right]^2},$$

where $y^h\left(b_t^j\right)$ is the model-implied yield corresponding to bond h and y_t^h is the market yield. To get a measure of global illiquidity, we construct a stock market-capitalization weighted average of each country-level illiquidity measure in line with our theoretical model proposed in the previous section:

$$\text{Illiq}_t^G = \frac{1}{\text{total market cap}_t}\sum_{j=1}^{6}\text{market cap}_t^j \times \text{Illiq}_t^j.$$

Market Illiquidity Proxy. To control for market illiquidity phenomena, we compute Amihud (2002) measures using our international stock data set. In line with the literature, we add a constant to the Amihud measure and take logs to reduce the impact of outliers. The stock-level measure is defined as:

$$\text{Amihud}_t^k = \log\left(1 + \sum_{t=1}^{T}\frac{|r_t^k|}{\text{vol}_t^k}\right),$$

where $|r_t^k|$ is the absolute return of stock k on day t, vol_t^k is the trading volume in the local currency of stock k on day t, obtained by multiplying the number of shares traded by the closing price, and T corresponds to the length of the window. Similar to Karolyi, Lee, and van Dijk (2012), we calculate Amihud_t^k for each stock based on daily data over a non-overlapping three-month rolling window. We first restrict the sample to stocks from major exchanges, except for Japan, where we use data from two exchanges (Osaka and Tokyo). We require that a stock has at least 10 valid daily observations (return and volume) during the three months. We delete stock days where the trading volume is below USD 100 and remove extreme observations manually. We use data from 1990 onwards, except for Germany, where we use data after 1999, because the daily trading volume is not available for most German stocks before that date. To get a country-level measure, we take a market capitalization weighted average for each stock.

[15] We thank Fulvio Pegoraro and Luca Tiozzo 'Pezzoli' for sharing their codes.

3 Illiquidity Facts

In this section we document the key time series and cross-sectional properties of our country and global illiquidity measures.

The time-series of all country illiquidity measures, in basis points, are plotted on Figure 2. Panel A of Table 2 reports summary statistics. The average pricing errors across all bonds in a given country and all periods range from 2.6 basis points (bps) for Japan to 7.7bps for Switzerland. This implies that deviations for some bonds in some periods are larger than these average values, representing substantial trading opportunities.[16]

To put the magnitude of these pricing deviations into economic perspective, we compare them to bid-ask spreads in government bond markets, a typical measure of market illiquidity. In the US Treasuries market, for which we have detailed bid-ask spread studies, pricing errors are larger than the bid-ask spread on average, and in particular are several times larger during illiquidity spikes. For instance, Engle, Fleming, Ghysels, and Nguyen (2013) report spreads for five-year Treasuries that do not exceed 1bps in normal times and 2bps at the height of the crisis in 2009.[17] Similarly, according to the Bank for International Settlements (2016) bid-ask spreads on German and Japanese ten-year bonds are below 2bps since 2008.

One might suspect that in the time-series, countries which tend to be more illiquid remain illiquid in the cross-section over long periods of time. To check this, at the end of each month we sort the country illiquidity proxies into terciles. Panel B reports the fraction of time each country illiquidity measure is in the low, medium, or high bin. The frequency of a country being in either of the three bins is almost equally distributed, with the exception of Japan, which is in the low illiquidity bin 2/3 of the time, and Switzerland, which is in the high illiquidity bin 74% of the time.

[Insert Table 2 and Figures 2 and 3 here.]

[16]To understand the size of these deviations, we can relate these numbers to other arbitrage strategies. For example, Pasquariello (2014) finds that the average violation from currency triangular arbitrage is 0.14bps, Rime, Schrimpf, and Syrstad (2017) document covered interest rate parity violations in the order of 3bps to 12bps for the countries we study (vis-à-vis the US$). Moreover in fixed income markets, even tiny deviations from arbitrage can still lead to significant arbitrage opportunities as for example in swap spread trading strategies (see, e.g., Duarte, Longstaff, and Yu (2007) for a discussion of fixed income arbitrage trading strategies).

[17]Dudley (2016) shows that bid-ask spreads on Treasury bond yields in general show very little variation and only spiked in 2008/2009 and have remained very stable since.

The time-series variation in country illiquidity exhibits significant commonality. Pairwise correlations between illiquidity proxies reported in Panel C of Table 2 are all positive and range between 20% (US and Japan) and 64% (Germany and Switzerland). Panel D of Table 2 reports loadings from the following regression:

$$\text{Illiq}_t^j = \beta_0^j + \beta_1^j \text{Illiq}_t^G + \epsilon_t^j,$$

where Illiq_t^j is the illiquidity proxy of country j and Illiq_t^G is the global illiquidity proxy. Unsurprisingly, we find that all country measures co-move positively with the global illiquidity factor, and that the latter explains a significant proportion of the variation in the country illiquidity, with R^2 ranging between 31% and 60%. At the same time, the high unconditional correlation between country-level illiquidities is driven by a few crisis episodes. The upper panel of Figure 4 plots the average conditional correlation among the different illiquidity proxies, calculated on daily data using a three-year rolling window. The average correlation peaks during periods of distress such as the dotcom bubble burst or the most recent financial crisis where the correlation reaches almost 80%, but is significantly lower otherwise.

[Insert Figure 4 here.]

Next, we look at the dispersion of illiquidity across countries. As seen in Table 2, the levels of country illiquidity are relatively close on average. In other words, there are no large permanent differences in illiquidity across the countries we consider. This is perhaps not surprising given that we include only developed financial markets in our analysis. However, illiquidity can become significantly dispersed when some countries experience idiosyncratic illiquidity episodes. The lower panel in Figure 4 illustrates the cross-sectional standard deviation of illiquidity measures.

Overall, illiquidity exhibits significant country variation: country- or region-specific events seem to be reflected in spikes in the respective local illiquidity measures that are not shared globally. For example, the Japanese measure is highly volatile in the early 1990s and around the Asian crisis of 1996–1997. It displays a more modest spike around the dot-com bubble burst in 2001 and during the most recent financial crisis. The German illiquidity proxy is especially volatile after 1992 and during the most recent financial crisis. The heightened level of the illiquidity proxy after 1990 can be explained by the large uncertainty surrounding the German reunification in October 1990 and the ERM crisis in September 1992. The repercussions of the ERM crisis are also found in the illiquidity proxies of the UK and Switzerland, where we see large jumps during the year 1992. Interestingly, these stark movements are completely absent in the US illiquidity proxy, which displays only moderate movements until 1997 Asian crisis, except around the first Gulf War in 1991. The downgrade of General Motors and Ford in May 2005, on the other hand, is a US-specific event, which is not reflected in the other five country illiquidity proxies.

4 Empirical Results

In this section we use the illiquidity measures to test our theoretical predictions. We first study the effect of the illiquidity level on the cross-section of international stock returns and then estimate the price of the illiquidity risk.

4.1 Global Illiquidity Level and the Security Market Line

Proposition 1 states that the slope of the average SML should depend negatively on the tightness of global funding constraints, while the intercept should be positively related to it. As a first illustration, we follow Cohen, Polk, and Vuolteenaho (2005) and divide our monthly data sample into quintiles according to the level of global illiquidity. We then examine the pricing of beta-sorted portfolios in these quintiles and estimate the empirical SML. Figure 5 depicts the average intercept and slope of the SML for different levels of global illiquidity ranging from low illiquidity (bin 1) to high illiquidity (bin 5).

[Insert Figure 5 here.]

In line with our prediction, the slope coefficient is decreasing with global illiquidity, while the intercept is increasing. For example, for low illiquidity states the average intercept is 0.025% with a slope of equal size, whereas for high illiquidity, the intercept increases to 0.180% and the slope decreases to -0.150%. The difference between the low and high illiquidity bin intercept is 0.205% per month, which is statistically different from zero with a t-statistic of 2.50. Similarly, the difference in slope coefficients, which is 0.175%, is highly statistically different from zero with a t-statistic of 4.21.

To test the proposition more formally, we follow Hong and Sraer (2016) and regress excess returns on a constant and the portfolios' trailing-window post-ranking beta:

$$rx_t^j = \text{intercept}_t + \text{slope}_t \times \beta_t^j + \epsilon_t^j,$$

where rx_t^j is the excess return of the j-th β-sorted portfolio and β_t^j is the post-ranking beta of portfolio j. This gives us the time-series of the intercept and the slope of the global SML for each quintile of global illiquidity. In the second stage, we estimate the following two regressions:

$$\text{intercept}_t = a_1 + b_1 \text{Illiq}_{t-1}^G + c_1 X_t + u_{1,t}, \quad (20)$$

$$\text{slope}_t = a_2 + b_2 \text{Illiq}_{t-1}^G + c_2 X_t + u_{2,t}, \quad (21)$$

where the vector X_t contains the controls such as the excess returns on the global market portfolio, the global size, value and momentum portfolio returns, as well as the global market volatility. We control for these variables as they may have an effect on the shape of the SML (see, e.g., Hong and Sraer (2016)). The estimated coefficients are presented in Table 3.

[Insert Table 3 here.]

In line with our theoretical predictions, we find that global illiquidity has a positive (negative) effect on the intercept (slope) of the SML. In univariate regressions, we find that the estimated coefficient on global illiquidity for the intercept (slope) is 0.009 (-0.019) with an associated t-statistic of 2.10 (-2.69). The coefficient and its statistical significance remain virtually unchanged when we control for the market, size, value, momentum, and market volatility factors.

4.2 Country Illiquidity and Alpha

We now inspect how returns vary in the cross-section of illiquidity and beta-sorted stocks. More specifically, Proposition 2 states that holding local illiquidity constant, a higher beta means lower alpha; holding beta constant, alpha increases in the local illiquidity. Panel A of Table 4 reports the results using our international stock data set. We consider three beta- and two illiquidity-sorted portfolios and document their average excess returns, alphas, market betas, volatilities, and Sharpe ratios. Consistent with the results in Frazzini and Pedersen (2013), we find that alphas decline from the low-beta to the high-beta portfolio: for low illiquidity stocks, the alpha decreases from 0.794% to 0.349%, and similarly, Sharpe ratios drop from 0.66 to 0.19. In the high illiquidity bin, we find a similar pattern. For example, high-beta stocks have average returns of 0.799% per month, while low-beta stocks feature a monthly return of 0.897%. Our theoretical predictions also concern the effect across the different illiquidity bins and indeed, we find an increase in average returns for all beta-bins as we move from low to high illiquidity stocks. For example, keeping betas constant, we find that alphas increase from the low illiquidity stocks to high illiquidity stocks: The alpha for low beta stocks increases from 0.794% per month to 0.854%, for medium beta it increases from 0.578% to 0.759%, and for high beta stock it increases from 0.349% to 0.741%.

[Insert Table 4 here.]

Using these results, we can now inspect the effect of country illiquidity on BAB strategies. Proposition 3 states that, everything else being equal, a BAB strategy should perform better in countries with higher local illiquidity. In order to test this prediction, we construct a BAB strategy within each country, and for every month sort the country BAB strategies into high and low illiquidity bins. The summary statistics of the two trading strategies are reported in Panel A of Table 5.

[Insert Table 5 here.]

We find that the high-illiquidity BAB portfolio produces significantly higher excess returns than a corresponding low-illiquidity BAB portfolio: The average monthly return on the former is 1.368% (with an associated t-statistic of 5.23) whereas the latter has an average return of 0.598% (with a t-statistic of 3.30). The alpha of the high illiquidity portfolio is 1.46% and the annualized Sharpe ratio is 1.07. If we would construct a high illiquidity minus low illiquidity portfolio (HML portfolio), we would have earned a monthly alpha of 0.854% with a t-statistic of 3.38, and an annualized Sharpe ratio of 0.61.

4.3 The Market Price of Illiquidity Risk

Our previous results highlight the role of illiquidity level. In the following, we want to study the effect of illiquidity risk. We ask whether the expected return of a stock is related to the sensitivity of its return to innovations in global illiquidity, and we estimate the market price of illiquidity risk using cross-sectional regressions. Theorem 3 states that global illiquidity should be a priced risk factor in the cross-section of stock returns in addition to the global market factor. More specifically, our model predicts the following relationship between excess returns, rx^j_{t+1}, and priced risk factors:

$$rx^j_{t+1} = a_t + b^j_{M,t} rx^G_{t+1} + b^j_{I,t} \Delta \text{Illiq}_{t+1} + \epsilon^j_{t+1},$$

where ΔIlliq_{t+1} is the innovation in global illiquidity and rx^G_{t+1} is the excess return on the global index. To calculate innovations, we first difference our global illiquidity proxy, using the following AR(1) specification:

$$\text{Illiq}_{t+1} = \rho \text{Illiq}_t + v_{t+1},$$

and use the innovations, v_{t+1}, as our measure of illiquidity ΔIlliq_{t+1}. As test assets, we use 5×5 market and global illiquidity-sorted stock returns. We summarize our results in Table 6 (Panel A). Recall from our theoretical model that the sign of the market price of illiquidity risk depends on investors' hedging demand components. On one hand, investors hedge against states of the world where the shadow price of an additional dollar of funding is high. On the other hand, investors also face standard intertemporal hedging demands for assets that pay off when markets are low. Since these effects move in opposite directions, the sign and magnitude of the illiquidity risk premium depend on the relative strength of the two hedging motives. In the data, we find that illiquidity is significantly priced: The estimated monthly premium is

-43 bps with a t-statistic of -3.52. This translates to an annualized illiquidity risk premium of about -5% per year, which is comparable in magnitude to Pástor and Stambaugh (2003) who find a market illiquidity risk premium of about 7% in US stocks.

[Insert Table 6 here.]

4.4 Robustness

Funding illiquidity and market illiquidity. A natural question is how and whether market illiquidity affects our results, and we explore whether our proxy of funding illiquidity is subsumed by measures of market illiquidity. To measure stock market liquidity, we use the Amihud (2002) measure.[18] We find that in general the unconditional pairwise correlation between our country-level illiquidity proxies and the corresponding country-level Amihud (2002) measure is not particularly high, ranging between 10% (Germany) to 40% (US).[19] Moreover, these correlations are mainly driven by the 2008/2009 global recession. To study the impact of funding illiquidity beyond market illiquidity, we regress our illiquidity measure for each country onto the country-level stock market illiquidity computed following Amihud (2002), and take the residual to be our new illiquidity measure, labelled Illiquidity$^{\perp}$. We then re-run our previous analysis using this new measure of illiquidity.

Panel B of Table 4 reports summary statistics of portfolios sorted on Illiquidity$^{\perp}$ and beta. Compared to our baseline results in Panel A of Table 4, we notice that the pattern remains exactly the same. In line with our theoretical prediction, when holding beta constant, average excess returns are higher when moving from low illiquidity to high illiquidity stocks. Similarly, alphas increase from low to high illiquidity portfolios and are statistically different from zero for all portfolios except the high beta, low illiquidity one. Panel B of Table 5 reports portfolio sorts of BAB strategies based on country-level illiquidity. We find that average returns, alphas

[18] We use the Amihud (2002) measure for several reasons. First, daily data is readily available for the countries we study. Second, Amihud, Hameed, Kang, and Zhang (2015) and Karolyi, Lee, and van Dijk (2017), among others, document the effect of Amihud (2002) market illiquidity on international stock returns. Other possible measures include the Pástor and Stambaugh (2003) Gamma, the Zero measure by Lesmond, Ogden, and Trzcinka (1999) and the Hasbrouck (2004) Gibbs measure. Goyenko, Holden, and Trzcinka (2009) and Fong, Holden, and Trzcinka (2011) run horse races among different liquidity proxies and recommend the Amihud (2002) measure as a good proxy of market illiquidity.

[19] Correlations in changes are lower and even negative in the case of Japan.

and Sharpe ratios are very close to the ones reported in Panel A of Table 5. The low (high) illiquidity portfolio yields a CAPM alpha of 63bps (150bps) per month with an associated t-statistic of 3.11 (3.60). We conclude that the effect of funding illiquidity as proxied by deviations from government bond prices is distinct and not subsumed by the market illiquidity measure of Amihud (2002).

Beta calculation. A large literature concludes that firm-specific betas are notoriously difficult to estimate due to measurement error. The typical approach to reduce measurement error in betas is to form portfolios as for example in Fama and MacBeth (1973). Another way to shrink beta estimates is toward an economically informative prior. The idea is that by incorporating prior cross-sectional information, one can increase the precision of the beta estimate. In this vein, Vasicek (1973) argues that one should adjust sample estimates toward a cross-sectional mean of one. Karolyi (1992), however, notes that this approach ignores firm-specific information, which is known prior to the estimation, and proposes to form industry portfolios and to shrink a portfolio's beta estimate toward its industry beta. In the following, we use the method of Karolyi (1992) and repeat our main analysis using these betas. Industry specifications for our countries are available from Datastream. A priori, however, we do not expect our results to change much: While it is possible that, for example, returns on betting-against-beta strategies are quantitatively different under different methods, our main focus is on how these strategies perform in low versus high illiquidity countries (see Proposition 2).[20]

Panel C of Table 4 summarizes portfolio returns of illiquidity-to-beta sorted portfolios. We find the results to be very similar to those reported in Panel A of Table 4 albeit average returns are slightly higher for the Karolyi (1992)-beta sorted portfolios. Similarly, Panel C of Table 5 reports sorts based on country-level BABs. Again, the results are qualitatively the same as in Panel A of Table 5 where we calculate betas based on Vasicek (1973) shrinking. If anything, returns and statistical significance are higher than compared to the previous results. For example, the high illiquidity BAB portfolio produces an average return of more than 163bps per month (t-statistic of 6.02) and an annualized Sharpe ratio of 1.24. We conclude that using betas according to Karolyi (1992) does not change our results.

[20]Indeed, we find Sharpe ratios of country-level BAB strategies to be higher when using the Karolyi (1992) method. However, the illiquidity ranking, i.e., whether a country is considered a low or high illiquidity country, is unaffected by the way we calculate betas.

Disagreement and short sale constraints. Frictions other than funding constraints could potentially affect the cross-section of expected returns. Hong and Sraer (2016), for example, argue that investors' disagreement combined with short sale constraints make the security market line flatter. According to the authors, high beta assets are more prone to speculation than low beta stocks and are more sensitive to aggregate disagreement about the stock market. Short-sale constraints then result in these high beta assets being overpriced. The authors use forecast dispersion of firms' earnings as a measure of aggregate disagreement. Since international earning forecasts are not available, we calculate a U.S. aggregate disagreement measure from individual firm-level disagreement measures multiplied by the firms' market capitalization. In line with Hong and Sraer (2016), we calculate disagreement as the cross-sectional standard deviation of stock market earning forecasts. We then run the following regression:

$$\begin{aligned} \text{intercept}_t &= a_1 + b_1 \text{Illiq}^G_{t-1} + c_1 \text{Illiq}^G_{t-1} \times \text{disagreement}_{t-1} + d_1 \text{disagreement}_{t-1} + e_1 r^G_t + u_{1,t}, \\ \text{slope}_t &= a_2 + b_2 \text{Illiq}^G_{t-1} + c_2 \text{Illiq}^G_{t-1} \times \text{disagreement}_{t-1} + d_2 \text{disagreement}_{t-1} + e_2 r^G_t + u_{2,t} \end{aligned}$$

The results are presented in the last two columns of each panel in Table 3. First, we find that illiquidity remains significant both the intercept and the slope of the SML with the coefficients virtually unchanged from our baseline results. Second, the disagreement proxy is not significant in either regressions. Finally, if the relation between illiquidity and the SML arises in part because illiquidity proxies for the severity of short sale constraints, the relation should be stronger during periods of high aggregate disagreement. We find that the interaction term is significant for the slope regression and has the expected negative sign, however it is not significant for the intercept regression. We conclude that while there is some indication that higher disagreement can lead to a lower slope of the security market line, the statistical evidence is not cogent. At the same time, evidence strongly supports the direct effect of illiquidity on the SML shape.

Omitted Factors. A fundamental concern when estimating risk premia is the omission of factors since for the estimation to correctly recover the risk premia, all priced risk factors in the economy need to be accounted for. While our theoretical model is based on two priced risk factors (the market portfolio and global illiquidity), in practice, there are obviously other factors that affect the cross-section of international stock returns. It is well known, that this omitted variable problem not only affects the size and significance of the estimated risk premia but also its sign. The standard approach of addressing the omitted variable bias, is to augment the regression with additional factors known to be priced. In the following, we address this issue in two ways. First, we estimate the price of global illiquidity risk using the omitted variable approach in Giglio and Xiu (2017). Second, we control for additional factors in our cross-sectional regressions.

Giglio and Xiu (2017) propose a three-pass regression method that combines principal components analysis with the two-stage Fama and MacBeth (1973) regression framework to estimate consistent factor risk premia in the presence of omitted factors. The main idea is that even if the true risk factors that drive equity returns are not known, the factors extracted using the principal component analysis are equally effective in controlling for the omitted risk factors for the purpose of estimating the risk premium of the factor of interest.

We present the results in Panel B of Table 6. We first notice that the sign and size of the market risk premium changes compared to the standard Fama and MacBeth estimation (see Panel A). The negative risk premium for market risk is a well-known result in the literature and points toward misspecification as for tradable factors the cross-sectional estimate of the risk premium should correspond to the time-series estimate of the average excess return of the portfolio. The three-pass method yields a positive risk premium estimate for the market factor. More importantly, we find that the estimated price of global illiquidity risk remains highly statistically significant (with an associated t-statistic of -3.09) and negative. The size drops from 43bps to 18bps, however. When we add other factors such as the global size, book-to-market, or momentum, we find none of the estimates to be statistically different from zero, moreover, the estimated coefficient and high statistical significance for global illiquidity risk remains the same.

Other robustness checks. Additional unreported checks support the robustness of our results to a range of empirical modelling choices. First, as an alternative to Svensson (1994), we use the Nelson and Siegel (1987) and a cubic spline methods to fit the yield curves. All three approaches lead to very similar results. We chose the Svensson (1994) method as our baseline over the other two because it is the most widely used and also the most flexible. Second, we note that the unconditional correlation between the market capitalization weighted average and the first principal component of our country illiquidity proxies is 95%. Using either of them leads to very similar results. We opt for the former measure of global illiquidity as it is in line with our model. Third, we verify that standardized illiquidity measures, i.e., we de-mean and divide by the standard deviation, result in a similar ranking of countries compared to non-standardized measures.

5 Conclusion

This paper investigates the effect of capital constraints on asset returns across different countries. We construct daily country-specific illiquidity proxies from pricing deviations on government bonds. While the overall correlation between the country-specific measures is high, the measures display distinct idiosyncratic behavior especially during country-specific political or economic events. The average level of illiquidity and the difference in illiquidity level across countries have an important effect on asset prices. In line with the prediction of the international CAPM with funding constraints, higher global illiquidity affects the international risk-return trade-off by lowering the slope and increasing the intercept of the average international security market line. In the same way, differences in country illiquidity are associated with significant differences in alpha: trading strategies that condition on illiquidity yield attractive returns with highly significant alpha and Sharpe ratios. Finally, we also show that not only the level of illiquidity matters but so does illiquidity risk. Using cross-sectional regressions, we find the price of the global illiquidity risk to be 5% per year, meaning that investors are willing to pay a significant premium to hedge against the deterioration of global funding conditions.

Our country illiquidity proxies can be used in several related avenues. First, idiosyncratic variation in the cross-section of illiquidity could be applied to test market segmentation. Second, our model is silent on countries' default risk, a paramount aspect in relation to the recent Eurozone crisis. It would be interesting to extend our model to feature credit risk to study the feedback between sovereign risk and illiquidity and its effect on asset prices. Finally, fixed income liquidity plays a pivotal role not just for the stability of the financial system but also for the conduct of monetary policy. Central banks therefore have a large interest in monitoring fixed income liquidity especially during market stress. For example, while the common understanding is that unconventional monetary policies in the current low-yield environment have supported bond valuations and reduced volatility in fixed income markets, we still observe large variability in our funding proxies post 2009. Modeling and analyzing the interaction between monetary policy in the context of funding illiquidity and its effect on asset prices is an important and challenging topic that we leave for future research.

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Appendix A Data

Appendix A.1 Bond Data

We apply the following data filters in order to account for institutional differences across countries and obtain securities with similar characteristics. In particular: (i) We exclude bonds with option like features such as bonds with warrants, floating rate bonds, callable and index-linked bonds. (ii) We consider only securities with a maturity of more than one year at issue and exclude securities that have a remaining maturity of less than three months. This is to alleviate concerns that short maturities are disconnected from the rest of the yield curve.[21] (iii) We exclude bonds with a remaining maturity of 15 years or more as in an international context they are often not very actively traded (see, e.g., Pegoraro, Siegel, and Tiozzo 'Pezzoli' (2013)). (iv) For the US we exclude the on-the-run and first-off-the-run issues for every maturity. These securities often trade at a premium to other Treasury securities. However, this premium may be in part explained by the higher market liquidity of the on-the-run issues and the severity of search frictions in the bond market. In addition, the on-the-run phenomenon is not present in other countries (see, e.g., Ejsing and Sihvonen (2009)), making the comparisons with the U.S. more difficult if on-the-run issues are kept for the U.S. (v) We exclude bonds if the reported prices are obviously wrong.

Appendix A.2 Stock Data and Betas

We only select stocks from major exchanges, which are defined as those in which the majority of stocks for a given country are traded. We exclude preferred stocks, depository receipts, real estate investment trusts, and other financial assets with special features based on the specific Datastream type classification. To limit the effect of survivorship bias, we include dead stocks in the sample. We exclude non-trading days, defined as days on which 90% or more of the stocks that are listed on a given exchange have a return equal to zero. We also exclude a stock if the number of zero-return days is more than 80% in a given month.

We follow Frazzini and Pedersen (2013) to construct ex-ante betas for our dataset of international stocks from rolling regressions of daily excess returns on market excess returns. The estimated beta for stock k at time t is given by:

$$\hat{\beta}^k_{t,\mathrm{TS}} = \hat{\rho}^k_t \frac{\hat{\sigma}^k_t}{\hat{\sigma}^G_t},$$

where $\hat{\sigma}^k_t$ and $\hat{\sigma}^G_t$ are the estimated volatilities for the stock and the market and $\hat{\rho}^k_t$ is their correlation. Volatilities and correlations are estimated separately. First, we use a one-year rolling standard deviation for volatilities and a five-year horizon for the correlation to account for the fact that correlations appear to move more slowly than volatilities. To account for non-synchronous trading, we use one-day log returns to estimate volatilities and three-day log returns for correlation. Finally, we shrink the time-series estimate of the beta towards the cross-sectional mean ($\beta^k_{t,\mathrm{CS}}$) following Vasicek (1973):

$$\hat{\beta}^k_t = \omega \hat{\beta}^k_{t,\mathrm{TS}} + (1-\omega)\,\hat{\beta}^k_{t,\mathrm{CS}},$$

where we set $\omega = 0.6$ for all periods and all stocks, in line with Frazzini and Pedersen (2013).

[21] Duffee (1996), for example, shows that Treasury bills exhibit a lot of idiosyncratic variation and have become increasingly disconnected from the rest of the yield curve.

Appendix A.3 Svensson (1994) Method

The Svensson (1994) model assumes that the instantaneous forward rate is given by

$$f_{m,b} = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp\left(-\frac{m}{\tau_1}\right) + \beta_3 \frac{m}{\tau_2} \exp\left(-\frac{m}{\tau_2}\right),$$

where m denotes the time to maturity and $b = (\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2)$ are parameters to be estimated. By integrating the forward rate curve, we derive the zero-coupon spot curve:

$$\begin{aligned} s_{m,b} \quad = \quad & \beta_0 + \beta_1 \left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right)\left(\frac{m}{\tau_1}\right)^{-1} \\ & + \beta_2 \left(\left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right)\left(\frac{m}{\tau_1}\right)^{-1} - \exp\left(-\frac{m}{\tau_1}\right)\right) \\ & + \beta_3 \left(\left(1 - \exp\left(-\frac{m}{\tau_2}\right)\right)\left(\frac{m}{\tau_2}\right)^{-1} - \exp\left(-\frac{m}{\tau_2}\right)\right). \end{aligned}$$

A proper set of parameter restrictions is given by $\beta_0 > 0, \beta_0 + \beta_1 > 0$, $\tau_1 > 0$, and $\tau_2 > 0$. For long maturities, the spot and forward rates approach asymptotically β_0, hence the value has to be positive. $(\beta_0 + \beta_1)$ determines the starting value of the curve at maturity zero. (β_2, τ_1) and (β_3, τ_2) determine the humps of the forward curve. The hump's magnitude is given by the absolute size of β_2 and β_3 while its direction is given by the sign. Finally, τ_1 and τ_2 determine the position of the humps.

Appendix B Proofs and derivations

Proof of Lemma 1. We substitute (4) into (2) to obtain wealth dynamics

$$W_{i,t} = W_{i,t}\left(1 + r_t\right) + \sum_{k\in\mathcal{K}} x_{i,t}^k \left(\mu_t^k - r_t + \left(\sigma_t^k\right)^\top \varepsilon_{t+1}\right). \tag{A-1}$$

Combining (3), (1), and (A-1), and denoting the Lagrange multiplier of (3) by $\psi_{i,t}$, the constrained optimization problem is equivalent to the following problem:

$$\max_{\left\{x_{i,t}^k\right\}_{k\in\mathcal{K}}} \sum_{k\in\mathcal{K}} x_{i,t}^k \left(\mu_t^k - r_t\right) - \frac{\alpha}{2}\left(\sum_{k\in\mathcal{K}} x_{i,t}^k \sigma_t^k\right)^\top \left(\sum_{k\in\mathcal{K}} x_{i,t}^k \sigma_t^k\right) - \psi_{i,t}\left[\sum_{k\in\mathcal{K}} m_{i,t}^k \left|x_{i,t}^k\right| - W_{i,t}\right].$$

Pointwise differentiation with respect to $x_{i,t}^k$ then yields the first-order conditions (5). □

Proof of Theorem 1. From (4) and (8), we write $R_{t+1}^G = \mu_t^G + \left(\sigma_t^G\right)^\top \varepsilon_{t+1}$ with

$$\mu_t^G = \frac{1}{G_t}\sum_{k\in\mathcal{K}} \theta_t^k \mu_t^k \text{ and } \sigma_t^G = \frac{1}{G_t}\sum_{k\in\mathcal{K}} \theta_t^k \sigma_t^k.$$

Thus,

$$\mu_t^G - r_t = \frac{1}{G_t}\sum_{k\in\mathcal{K}} \theta_t^k\left(\mu_t^k - r_t\right) = \frac{1}{|\mathcal{I}|\,G_t}\left[\alpha\left(\sum_{k\in\mathcal{K}} \theta_t^k \sigma_t^k\right)^\top \left(\sum_{k\in\mathcal{K}} \theta_t^k \sigma_t^k\right) + \sum_{k\in\mathcal{K}} \theta_t^k \sum_{i\in\mathcal{I}} \psi_{i,t} m_{i,t}^k \operatorname{sgn}\left(x_{i,t}^k\right)\right].$$

Moreover, we have

$$\operatorname{Cov}_t\left[R_{t+1}^k, R_{t+1}^G\right] = \left(\sigma_t^k\right)^\top \sigma_t^G = \frac{1}{G_t}\left(\sigma_t^k\right)^\top \sum_{k'\in\mathcal{K}} \theta_t^{k'} \sigma_t^{k'}$$

and

$$\operatorname{Var}_t\left[R_{t+1}^G\right] = \left(\sigma_t^G\right)^\top \sigma_t^G = \frac{1}{G_t^2}\left(\sum_{k\in\mathcal{K}} \theta_t^k \sigma_t^k\right)^\top \sum_{k\in\mathcal{K}} \theta_t^k \sigma_t^k.$$

From here, after some algebra (7) becomes

$$\begin{aligned}
\mu_t^k - r_t &= \frac{\alpha}{|\mathcal{I}|}\left(\sigma_t^k\right)^\top \sum_{k'\in\mathcal{K}} \theta_t^{k'} \sigma_t^{k'} + \frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}} \psi_{i,t} m_{i,t}^k \operatorname{sgn}\left(x_{i,t}^k\right) \\
&= \beta_t^k\left(\mu_t^G - r_t\right) - \beta_t^k \sum_{k\in\mathcal{K}} \frac{1}{G_t}\theta_t^k\left[\frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}} \psi_{i,t} m_{i,t}^k \operatorname{sgn}\left(x_{i,t}^k\right)\right] + \frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}} \psi_{i,t} m_{i,t}^k \operatorname{sgn}\left(x_{i,t}^k\right) \\
&= \beta_t^k\left(\mu_t^G - r_t\right) - \beta_t^k \sum_{k\in\mathcal{K}} \frac{1}{G_t}\theta_t^k \phi_t^k + \phi_t^k,
\end{aligned}$$

where in the last step we used (10). This completes the proof of Theorem 1. □

Proof of Theorem 2. Before providing the proof, we start by expressing bond yields in a general form to illustrate which ingredients are necessary and sufficient to obtain deviations from the smooth yield curve in equilibrium. Then we provide the proof in the specific case discussed by Theorem 2.

Lemma 3. *The equilibrium yield of bond h is given by*

$$y_t^{j,h} = \frac{1}{\tau_h}\left(\sum_{s=t}^{t+\tau_h-1} E_t\left[r_s\right] + \frac{\alpha}{|\mathcal{I}|}E_t\left[\sum_{s=t}^{t+\tau_h-2}\left(\sigma_s^{t+\tau_h-s}\right)^\top\left(\Omega_{\varepsilon\xi}^\top\sum_{k\in\mathcal{K}}\theta_s^k\sigma_s^k + \sum_{h'\in\mathcal{H}} d_s^{t+\tau_{h'}-s}\sigma_s^{t+\tau_{h'}-s}\right)\right]\right) \tag{A-2}$$

$$+\frac{1}{\tau_h}E_t\left[\sum_{s=t}^{t+\tau_h-2}\sum_{i\in\mathcal{I}}\frac{1}{|\mathcal{I}|}\psi_{i,s}m_{i,s}^{t+\tau_h-s}sgn\left(z_{i,s}^{t+\tau_h-s}\right)\right],$$

where with a slight abuse of notation $t+\tau_h-s$ for $s\geq t$ refers to the future 1-period return volatilities of bond h over its lifetime.

Proof. Adjusting the budget constraint (2) and the margin constraint (3) for bonds we get

$$W_{i,t} = W_{i,t}\left(1+r_t\right) + \sum_{k\in\mathcal{K}} x_{i,t}^k\left(R_{t+1}^k - r_t\right) + \sum_{j\in\mathcal{J}}\sum_{h\in\mathcal{H}_j} z_{i,t}^{j,h}\left(R_{t+1}^{j,h}-r_t\right), \tag{A-3}$$

and

$$\sum_{k\in\mathcal{K}} m_{i,t}^k\left|x_{i,t}^k\right| + \sum_{j\in\mathcal{J}}\sum_{h\in\mathcal{H}_j} m_{i,t}^{j,h}\left|z_{i,t}^{j,h}\right| \leq W_{i,t}, \tag{A-4}$$

where $R_{t+1}^{j,h} \equiv \tau_h y_{i,t}^{j,h} - (\tau_h-1)\, y_{i,t+1}^{j,h-1}$ denotes the one-period return on bond h in country j between t and $t+1$. We write bond returns in the form

$$R_{t+1}^{j,h} = \mu_t^{j,h} + \left(\sigma_t^{j,h}\right)^\top \xi_{t+1}, \tag{A-5}$$

where $\mu_t^{j,h}\in\mathbb{R}$ is the expected net return of the bond, $\sigma_t^{j,h}\in\mathbb{R}^M$ is an M-dimensional vector, ξ_{t+1} is an M-dimensional random vector with mean zero and the identity covariance matrix that collects all bond-relevant uncertainty, and thus $\mathrm{Cov}_t\left[R_{t+1}^{j,h}, R_{t+1}^{j',h'}\right] = \left(\sigma_t^{j,h}\right)^\top\sigma_t^{j',h'}$ is the covariance between the returns of two bonds, h and h', from two countries, j and j'. Further, let us denote the $N\times M$ covariance matrix between ε_{t+1} and ξ_{t+1} by $\mathrm{Cov}_t\left[\varepsilon_{t+1},\xi_{t+1}\right]\equiv\Omega_{\varepsilon\xi}$.

Substituting (A-5) into (2) and then into (1), and denoting the Lagrange multiplier of (A-4) by $\psi_{i,t}$, we obtain:

$$\max_{\{x_{i,t}^k\}_{k\in\mathcal{K}};\{x_{i,t}^h\}_{h\in\mathcal{H}}} \sum_{k\in\mathcal{K}} x_{i,t}^k\left(\mu_t^k - r_t\right) + \sum_{j\in\mathcal{J}}\sum_{h\in\mathcal{H}_j} z_{i,t}^{j,h}\left(\mu_t^{j,h}-r_t\right) - \frac{\alpha}{2}\left(\sum_{k\in\mathcal{K}} x_{i,t}^k\sigma_t^k\right)^\top\left(\sum_{k\in\mathcal{K}} x_{i,t}^k\sigma_t^k\right)$$
$$-\alpha\left(\sum_{k\in\mathcal{K}} x_{i,t}^k\sigma_t^k\right)^\top\Omega_{\varepsilon\xi}\left(\sum_{j\in\mathcal{J},h\in\mathcal{H}_j} z_{i,t}^{j,h}\sigma_t^{j,h}\right) - \frac{\alpha}{2}\left(\sum_{j\in\mathcal{J},h\in\mathcal{H}_j} z_{i,t}^{j,h}\sigma_t^{j,h}\right)^\top\left(\sum_{j\in\mathcal{J},h\in\mathcal{H}_j} z_{i,t}^{j,h}\sigma_t^{j,h}\right)$$
$$-\psi_{i,t}\left[\sum_{k\in\mathcal{K}} m_{i,t}^k\left|x_{i,t}^k\right| + \sum_{j\in\mathcal{J},h\in\mathcal{H}_j} m_{i,t}^{j,h}\left|z_{i,t}^{j,h}\right| - W_{i,t}\right].$$

The FOCs of this problem become

$$\mu_t^k - r_t = \alpha\left(\sigma_t^k\right)^\top\left[\sum_{k'\in\mathcal{K}} x_{i,t}^{k'}\sigma_t^{k'} + \Omega_{\varepsilon\xi}\sum_{j\in\mathcal{J},h'\in\mathcal{H}_j} z_{i,t}^{j,h'}\sigma_t^{j,h'}\right] + \psi_{i,t}m_{i,t}^k\mathrm{sgn}\left(x_{i,t}^k\right). \tag{A-6}$$

and

$$\mu_t^{j,h} - r_t = \alpha \left(\sigma_t^{j,h}\right)^\top \left[\Omega_{\varepsilon\xi}^\top \sum_{k' \in \mathcal{K}} x_{i,t}^{k'} \sigma_t^{k'} + \sum_{j \in \mathcal{J}, h' \in \mathcal{H}_j} z_{i,t}^{j,h'} \sigma_t^{j,h'}\right] + \psi_{i,t} m_{i,t}^{j,h} \mathrm{sgn}\left(z_{i,t}^{j,h}\right). \tag{A-7}$$

From (A-8), the expected return on bond (j,h) depends on its exposure to interest rate risk $\sigma_t^{j,h}$. In addition, similar to stocks, bonds for which investors tend to have a long (short) position will tend to be more expensive (cheap) to compensate investors for the capital they have to commit. Moreover, the magnitude of this effect depends on the capital requirement for the bond in question, $m_{i,t}^{j,h}$. Aggregating across agents and imposing market clearing, we obtain

$$\mu_t^{j,h} - r_t = \frac{\alpha}{|\mathcal{I}|} \left(\sigma_t^{j,h}\right)^\top \left[\Omega_{\varepsilon\xi}^\top \sum_{k \in \mathcal{K}} \theta_t^k \sigma_t^k + \sum_{j \in \mathcal{J}, h' \in \mathcal{H}_j} d_t^{j,h'} \sigma_t^{j,h'}\right] + \frac{1}{|\mathcal{I}|} \sum_{i \in \mathcal{I}} \psi_{i,t} m_{i,t}^{j,h} \mathrm{sgn}\left(z_{i,t}^{j,h}\right). \tag{A-8}$$

Using the definition of $R_{t+1}^{j,h}$, we then write

$$y_t^{j,h} = \frac{1}{\tau_h} \sum_{s=t+1}^{t+\tau_h} E_t\left[R_s^{j,t+\tau_h+1-s}\right] = \frac{1}{\tau_h}\left(\mu_t^{j,h} + \sum_{s=t+1}^{t+\tau_h-1} E_t\left[\mu_s^{j,t+\tau_h-s}\right]\right).$$

Substituting in (A-8) and grouping terms, we obtain (A-2). □

For the proof of Theorem 2, let us assume that $m_{i,t}^{j,h}$, $d_t^{j,h}$, $m_{i,t}^k$, θ_t^k and $W_{i,t}$ are all constant over time and that the short rate r_t follows (11). We conjecture and later verify that there exists an equilibrium in which each investor invests a constant proportion of her wealth into each bond and stock, and that in this equilibrium bond yields are in the form

$$y_t^{j,h} = \frac{A_{\tau_h}(\tau_h) + B_h(\tau_h) r_t}{h}, \tag{A-9}$$

where $A_h(1) = 0$ and $B_h(1) = 1$ for all h. First, it implies that $\Omega_{\varepsilon\xi}^\top = 0$. Second, rewriting (A-4) in the form

$$\sum_{k \in \mathcal{K}} m_{i,t}^k \left|\frac{x_{i,t}^k}{W_{i,t}}\right| + \sum_{j \in \mathcal{J}, h \in \mathcal{H}_j} m_{i,t}^{j,h} \left|\frac{z_{i,t}^{j,h}}{W_{i,t}}\right| \leq 1,$$

our assumption means that both the $\left|x_{i,t}^k / W_{i,t}\right|$ and $\left|z_{i,t}^{j,h} / W_{i,t}\right|$ ratios and all sgn $\left(z_{i,t}^{j,h}\right)$ are all constant over time, thus the LHS is also constant over time. Therefore, either the constraint never binds for investor i, with $\psi_{i,t} = 0$ for all t, or it always binds and we conjecture (and verify later) that it the same Lagrange multiplier $\psi_{i,t} = \psi_i > 0$ for all t. Third, combining (A-9) and the relationship between returns and yields, we have that

$$\begin{aligned} R_{t+1}^{j,h} &= A_h(\tau_h) + B_h(\tau_h) r_t - A_h(\tau_h - 1) - B_h(\tau_h - 1) r_{t+1} \\ &= \left[A_h(\tau_h) - A_h(\tau_h - 1) - \kappa \bar{r} B_h(\tau_h - 1)\right] + \left[B_h(\tau_h) - (1-\kappa) B_h(\tau_h - 1)\right] r_t - B_h(\tau_h - 1)\sigma\eta_{t+1}, \end{aligned}$$

i.e., $\sigma_t^{j,h} = B_h(\tau_h - 1)\sigma$, one-dimensional and constant over time, and

$$\mu_t^{j,h} = \left[A_h(\tau_h) - A_h(\tau_h - 1) - \kappa \bar{r} B_h(\tau_h - 1)\right] + \left[B_h(\tau_h) - (1-\kappa) B_h(\tau_h - 1)\right] r_t.$$

Substituting it back to (A-8) we obtain

$$\left[A_h(\tau_h) - A_h(\tau_h - 1) - \kappa\bar{r}B_h(\tau_h - 1)\right] + \left[B_h(\tau_h) - (1-\kappa)B_h(\tau_h - 1) - 1\right]r_t$$
$$= \frac{\alpha}{|\mathcal{I}|}B_h(\tau_h - 1)\,\sigma \sum_{j\in\mathcal{J},h'\in\mathcal{H}_j} B_h(\tau_h' - 1)\,\sigma d_t^{j,h'} + \frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}}\psi_{i,t}m_{i,t}^{j,h}\ \mathrm{sgn}\left(z_{i,t}^{j,h}\right).$$

The RHS in this equation is constant, so collecting r_t terms it must be that

$$B_h(\tau_h) - (1-\kappa)B_h(\tau_h - 1) - 1 = 0,$$

i.e., B_h is the standard discrete-time coefficient for each h and j:

$$B_h(\tau_h) = B(\tau_h) = \frac{1-(1-\kappa)^{\tau_h}}{\kappa}.$$

On the other hand, collecting constant terms and rearranging implies

$$A_h(\tau_h) - A_h(\tau_h - 1) = \left[\kappa\bar{r} + \frac{\alpha}{|\mathcal{I}|}\sigma\sum_{j\in\mathcal{J}}\sum_{h'\in\mathcal{H}_j}\sigma_t^{j,h'}d_t^{j,h'}\right]B(\tau_h - 1) + C_{j,h}(\tau_h) - C_{j,h}(\tau_h - 1), \quad \text{(A-10)}$$

where we introduce the function $C_{j,h}$ that satisfies $C_{j,h}(1) = 0$ and, for $\tau_h \geq 2$,

$$C_{j,h}(\tau_h) - C_{j,h}(\tau_h - 1) = \frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}}\psi_{i,t}m_{i,t}^{j,h}\mathrm{sgn}\left(z_{i,t}^{j,h}\right). \quad \text{(A-11)}$$

To obtain $A_h(\tau_h)$ and hence bond prices, we need to solve the ODE (A-10). Notice first, that if the constraint does not bind for any agent, $\psi_i > 0$ for all $i \in \mathcal{I}$, then $C_{j,h}(\tau_h) = 0$ for all maturities. In this case the RHS of (A-10) consists of only one term, which depends on maturity τ_h and the aggregate amount of interest rate risk in the economy, $\sum_{j\in\mathcal{J}}\sum_{h'\in\mathcal{H}_j}\sigma_t^{j,h'}d_t^{j,h'}$. The solution in this case is

$$A_h(\tau_h) = \bar{A}(\tau_h) = \left[\kappa\bar{r} + \frac{\alpha}{|\mathcal{I}|}\sigma\sum_{j\in\mathcal{J},h'\in\mathcal{H}_j}\sigma_t^{j,h'}d_t^{j,h'}\right]\frac{\kappa\tau_h - 1 + (1-\kappa)^{\tau_h}}{\kappa^2},$$

in line with the solution of standard affine models, and constant across countries.

On the other hand, if there exist some $i \in \mathcal{I}$ with $\psi_i > 0$, $C_{j,h}(\tau_h)$ is non-trivial, and depends on margins and asset positions; in fact, (A-11) implies

$$C_{j,h}(\tau_h) = \frac{1}{|\mathcal{I}|}\sum_{\substack{h'\in\mathcal{H}_j:\\ \tau_{h'}\leq\tau_h}}\sum_{i\in\mathcal{I}}\psi_i m_i^{j,h'}\mathrm{sgn}\left(z_i^{j,h'}\right),$$

and $A_h(\tau_h) = \bar{A}(\tau_h) + C_{j,h}(\tau_h)$. As *sgn* (.) is a function that is discontinuous at zero, $C_{j,h}(\tau_h)$ can create a non-smooth component in $A_h(\tau_h)$ that varies across countries and the maturity structure of an individual country, too. Defining

$$\mathcal{A}(\tau_h) = \frac{\bar{A}(\tau_h)}{\tau_h},\quad \mathcal{B}(\tau_h) = \frac{B(\tau_h)}{\tau_h},\ \text{and}\ \ \mathcal{C}_{j,h}(\tau_h) = \frac{C_{j,h}(\tau_h)}{\tau_h}$$

completes the proof of Theorem 2. □

Proof of Lemma 2. Note that for any random variable $W_{i,2}$ that is realized at time 2 we can write

$$Var_0[W_{i,2}] = E_0\left[W_{i,2}^2\right] - (E_0[W_{i,2}])^2 = E_0\left[E_1\left[W_{i,2}^2\right]\right] - (E_0[W_{i,2}])^2 = E_0\left[E_1\left[W_{i,2}^2\right] - (E_0[W_{i,2}])^2\right]$$
$$+ E_0\left[(E_1[W_{i,2}])^2\right] - (E_0[E_1[W_{i,2}]])^2 = E_0[Var_1[W_{i,2}]] + Var_0[E_1[W_{i,2}]].$$

Substituting the RHS into (12), after some algebra we obtain (16).

Based on our previous results, the solution to an old agent's optimization problem (15) subject to (13) and (14) at $t = 1$ satisfies

$$\mu_1^k = \alpha\left(\sigma_1^k\right)^\top \sum_{k' \in \mathcal{K}} x_{i,1}^{k'} \sigma_1^{k'} + \psi_{i,1} m_{i,1}^k \operatorname{sgn}\left(x_{i,1}^k\right). \tag{A-12}$$

Further, it is straightforward from the budget constraint at $t = 1$ and the envelope theorem that

$$\frac{\partial V_{i,1}}{\partial W_{i,1}} = 1 + \psi_{i,1}.$$

Combining it with (13) at $t = 0$, we obtain

$$\frac{\partial \mathrm{E}_0[V_{i,1}]}{\partial x_{i,0}^k} = \mathrm{E}_0\left[\frac{\partial V_{i,1}}{\partial W_{i,1}} \frac{\partial W_{i,1}}{\partial x_{i,0}^k}\right] = \mathrm{E}_0\left[(1+\psi_{i,1}) R_1^k\right] = (1 + \mathrm{E}_0[\psi_{i,1}])\mu_0^k + \mathrm{Cov}_0\left[R_1^k, \psi_{i,1}\right] \tag{A-13}$$
$$= (1 + \mathrm{E}_0[\psi_{i,1}])\mu_0^k + \ \mathrm{Cov}_0\left[R_1^k, \psi_{i,1}\right].$$

Next, using that for any variable x and random variable $Y(x)$

$$\frac{\partial \mathrm{Var}[Y]}{\partial x} = \mathrm{E}\left[\frac{\partial Y^2}{\partial x}\right] - \frac{\partial\left(\mathrm{E}^2[Y]\right)}{\partial x} = 2\mathrm{E}\left[Y\frac{\partial Y}{\partial x}\right] - 2\mathrm{E}[Y]\mathrm{E}\left[\frac{\partial Y}{\partial x}\right] = 2\mathrm{Cov}\left[Y, \frac{\partial Y}{\partial x}\right],$$

and the budget constraint (13) written at $t = 0$ and 1, we have

$$\frac{1}{2}\frac{\partial \mathrm{Var}_0[\mathrm{E}_1[W_{i,2}]]}{\partial x_{i,0}^k} = \mathrm{Cov}_0\left[\mathrm{E}_1[W_{i,2}], \frac{\partial \mathrm{E}_1[W_{i,2}]}{\partial x_{i,0}}\right] \tag{A-14}$$
$$= \mathrm{Cov}_0\left[W_{i,1} + \sum_{k\in\mathcal{K}} x_{i,1}^k \mu_1^k, \frac{\partial\left(W_{i,1} + \sum_{k\in\mathcal{K}} x_{i,1}^k \mu_1^k\right)}{\partial x_{i,0}}\right] = \mathrm{Cov}_0\left[W_{i,1} + \sum_{k\in\mathcal{K}} x_{i,1}^k \mu_1^k, (1+\xi_{i,1}) R_1^k\right],$$

where in the last line we used the definition of $\xi_{i,1}$ from Lemma 2. Therefore, when writing the Lagrangian of the constrained optimization problem (16) subject to the budget and leverage constraints at time $t = 0$,

$$\max_{\left\{x_{i,0}^k\right\}_{k\in\mathcal{K}}} \mathrm{E}_0[V_{i,1}] - \frac{\alpha}{2}\mathrm{Var}_0[\mathrm{E}_1[W_{i,2}]] - \psi_{i,0}\left(\sum_{k\in\mathcal{K}} m_{i,0}^k \left|x_{i,0}^k\right| - W_{i,0}\right),$$

the FOC is

$$0 = \frac{\partial \mathrm{E}_0[V_{i,1}]}{\partial x_{i,0}^k} - \frac{\alpha}{2}\frac{\partial \mathrm{Var}_t[\mathrm{E}_{t+1}[W_{i,2}]]}{\partial x_{i,0}^k} - \psi_{i,0} m_{i,0}^k \operatorname{sgn}\left(x_{i,0}^k\right).$$

Together with (A-13) and (A-14), after some algebra, this equation implies (17). □

Proof of Theorem 3. The proof consists of rewriting and aggregating the first order conditions (A-12) and (17) across all investors i, and imposing market clearing at $t = 1$ and 0.

We start by studying the term $\xi_{i,1}$. If the constraint (3) does not bind in equilibrium for agent i at date 1, then her optimal positions are unaffected by wealth and we have $\xi_{i,1} = \psi_{i,1} = 0$. Suppose now that the constraint binds and all agents take positive positions in equilibrium, i.e., the FOC becomes

$$\mu_1^k = \alpha \left(\sigma_1^k\right)^\top \sum_{k' \in \mathcal{K}} x_{i,1}^{k'} \sigma_1^{k'} + \psi_{i,1} m_{i,1}^k,$$

or, in matrix form:

$$\mu_1 = \alpha \Omega_{R_2} x_{i,1} + \psi_{i,1} m_{i,1},$$

where μ_1 is the (given) vector of expected excess returns, and Ω_{R_2} is the variance-covariance matrix of R_2 returns, whose (k, k') element is $\left(\sigma_1^k\right)^\top \sigma_1^{k'}$. Thus,

$$x_{i,1} = \frac{1}{\alpha} \Omega_{R_2}^{-1} \left(\mu_1 - \psi_{i,1} m_{i,1}\right).$$

If the constraint binds, we have

$$W_{i,1} = \sum_{k \in \mathcal{K}} m_{i,1}^k \left| x_{i,1}^k \right| = m_{i,1}^\top x_{i,1} = \frac{1}{\alpha} m_{i,1}^\top \Omega_{R_2}^{-1} \left(\mu_1 - \psi_{i,1} m_{i,1}\right), \tag{A-15}$$

so differentiating with respect to $W_{i,1}$, we obtain

$$1 = \frac{1}{\alpha} \frac{\partial}{\partial W_{i,1}} \left[m_{i,1}^\top \Omega_{R_2}^{-1} \left(\mu_1 - \psi_{i,1} m_{i,1}\right) \right] = -\frac{1}{\alpha} \frac{\partial \psi_{i,1}}{\partial W_{i,1}} m_{i,1}^\top \Omega_{R_2}^{-1} m_{i,1}, \tag{A-16}$$

i.e.,

$$\frac{\partial \psi_{i,1}}{\partial W_{i,1}} = -\frac{\alpha}{m_{i,1}^\top \Omega_{R_2}^{-1} m_{i,1}}. \tag{A-17}$$

Further,

$$\sum_{k \in \mathcal{K}} x_{i,1}^k \mu_1^k = x_{i,1}^\top \mu_1 = \left(\mu_1 - \psi_{i,1} m_{i,1}\right)^\top \Omega_{R_2}^{-1} \mu_1,$$

thus

$$\begin{aligned} \xi_{i,1} &= \frac{\partial \sum_{k \in \mathcal{K}} x_{i,1}^k \mu_1^k}{\partial W_{i,1}} = \frac{\partial \left(x_{i,1}^\top \mu_1 \right)}{\partial W_{i,1}} = \frac{1}{\alpha} \frac{\partial \left(\left(\mu_1 - \psi_{i,1} m_{i,1}\right)^\top \Omega_{R_2}^{-1} \mu_1 \right)}{\partial W_{i,1}} \\ &= -\frac{1}{\alpha} m_{i,1}^\top \Omega_{R_2}^{-1} \mu_1 \frac{\partial \psi_{i,1}}{\partial W_{i,1}} = \frac{m_{i,1}^\top \Omega_{R_2}^{-1} \mu_1}{m_{i,1}^\top \Omega_{R_2}^{-1} m_{i,1}}. \end{aligned} \tag{A-18}$$

Moreover, in equilibrium we have

$$\mu_1 = \alpha \frac{1}{|\mathcal{I}|} \Omega_{R_2} \theta_1 + \frac{1}{|\mathcal{I}|} \sum_{i \in \mathcal{I}} \psi_{i,1} m_{i,1}; \tag{A-19}$$

this, together with (A-18) implies

$$\xi_{i,1} = \alpha \frac{1}{|\mathcal{I}|} \frac{m_{i,1}^\top \theta_1}{m_{i,1}^\top \Omega_{R_2}^{-1} m_{i,1}} + \frac{1}{|\mathcal{I}|} \frac{m_{i,1}^\top \Omega_{R_2}^{-1} \left(\sum_{i \in \mathcal{I}} \psi_{i,1} m_{i,1} \right)}{m_{i,1}^\top \Omega_{R_2}^{-1} m_{i,1}} \tag{A-20}$$

for all i whose constraint binds in equilibrium. If $m_{i,1}$ is the same for all agents, $m_{i,1} = m_1$, and it is either deterministic or stochastic but independent from all other random variables, we can write $\sum_{i\in\mathcal{I}} \psi_{i,1} m_{i,1} = m_i \sum_{i\in\mathcal{I}} \psi_{i,1}$. Thus, from (A-20), $\xi_{i,1}$ is the same for all agents whose constraint binds. Let us for simplicity assume that it binds for all agents so $\xi_{i,1}$ is constant across all agents. Aggregating (17) across agents, we obtain

$$\left(1 + \mathrm{E}_0\left[\frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}}\psi_{i,1}\right]\right)\mu_0^k = -\mathrm{Cov}_0\left[R_1^k, \frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}}\psi_{i,1}\right] \tag{A-21}$$
$$+\alpha\frac{1}{|\mathcal{I}|}\mathrm{Cov}_0\left[\sum_{i\in\mathcal{I}} W_{i,1} + \sum_{i\in\mathcal{I}}\sum_{k'\in\mathcal{K}} x_{i,1}^{k'}\mu_1^{k'}, (1+\xi_{i,1})\, R_1^k\right] + \frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}}\psi_{i,0} m_{i,0}^k \mathrm{sgn}\left(x_{i,0}^k\right).$$

Due to market clearing at date 1, we have

$$\sum_{i\in\mathcal{I}}\sum_{k'\in\mathcal{K}} x_{i,1}^{k'}\mu_1^{k'} = \sum_{k'\in\mathcal{K}}\theta_1^{k'}\mu_1^{k'} = \alpha\frac{1}{|\mathcal{I}|}\theta_1^\top\Omega_{R_2}\theta_1 + \frac{1}{|\mathcal{I}|} m_1^\top\theta_1\sum_{i\in\mathcal{I}}\psi_{i,1},$$

and due to market clearing at date 0, we can write

$$\sum_{i\in\mathcal{I}} W_{i,1} = \sum_{i\in\mathcal{I}} W_{i,0} + \sum_{k'\in\mathcal{K}}\left(\sum_{i\in\mathcal{I}} x_{i,0}^{k'}\right) R_1^{k'} = \sum_{i\in\mathcal{I}} W_{i,0} + \sum_{k'\in\mathcal{K}}\theta_0^{k'} R_1^{k'}.$$

Thus,

$$\mathrm{Cov}_0\left[\sum_{i\in\mathcal{I}} W_{i,1} + \sum_{i\in\mathcal{I}}\sum_{k'\in\mathcal{K}} x_{i,1}^{k'}\mu_1^{k'}, (1+\xi_{i,1})\, R_1^k\right] =$$
$$= \mathrm{Cov}_0\left[\sum_{i\in\mathcal{I}} W_{i,0} + \sum_{k'\in\mathcal{K}}\theta_0^{k'} R_1^{k'} + \alpha\frac{1}{|\mathcal{I}|}\theta_1^\top\Omega_{R_2}\theta_1 + \frac{1}{|\mathcal{I}|} m_1^\top\theta_1\sum_{i\in\mathcal{I}}\psi_{i,1}, (1+\xi_{i,1})\, R_1^k\right]$$
$$= \mathrm{Cov}_0\left[\sum_{k'\in\mathcal{K}}\theta_0^{k'} R_1^{k'} + m_1^\top\theta_1\Psi_1, \left(1 + \alpha\frac{1}{|\mathcal{I}|}\frac{m_{i,1}^\top\theta_1}{m_{i,1}^\top\Omega_{R_2}^{-1} m_{i,1}} + \frac{m_{i,1}^\top\Omega_{R_2}^{-1} m_{i,1}}{m_{i,1}^\top\Omega_{R_2}^{-1} m_{i,1}}\Psi_1\right) R_1^k\right],$$

and hence (A-21) becomes

$$\left(1 + \mathrm{E}_0\left[\Psi_1\right]\right)\mu_0^k = -\mathrm{Cov}_0\left[R_1^k, \Psi_1\right] \tag{A-22}$$
$$+\alpha\frac{1}{|\mathcal{I}|}\mathrm{Cov}_0\left[G_0 R_1^G + m_1^\top\theta_1\Psi_1, \left(1 + \alpha\frac{1}{|\mathcal{I}|}\frac{m_{i,1}^\top\theta_1}{m_{i,1}^\top\Omega_{R_2}^{-1} m_{i,1}} + \frac{m_{i,1}^\top\Omega_{R_2}^{-1} m_{i,1}}{m_{i,1}^\top\Omega_{R_2}^{-1} m_{i,1}}\Psi_1\right) R_1^k\right] + \phi_0^k,$$

where we introduce the notation Ψ_1 for the average Lagrange multiplier at date 1, as written in (19), and ϕ_0^k is as in (10). Breaking up the big covariance in (A-22) and grouping them gives

$$\left(1 + \mathrm{E}_0\left[\Psi_1\right]\right)\mu_0^k = \left(\alpha\frac{1}{|\mathcal{I}|} m_1^\top\theta_1 - 1\right)\mathrm{Cov}_0\left[R_1^k, \Psi_1\right] + \alpha\frac{1}{|\mathcal{I}|} G_0\mathrm{Cov}_0\left[R_1^G, R_1^k\right] \tag{A-23}$$
$$+\alpha\frac{1}{|\mathcal{I}|}\mathrm{Cov}_0\left[G_0 R_1^G + m_1^\top\theta_1\Psi_1, \left(\alpha\frac{1}{|\mathcal{I}|}\frac{m_{i,1}^\top\theta_1}{m_{i,1}^\top\Omega_{R_2}^{-1} m_{i,1}} + \frac{m_{i,1}^\top\Omega_{R_2}^{-1} m_{i,1}}{m_{i,1}^\top\Omega_{R_2}^{-1} m_{i,1}}\Psi_1\right) R_1^k\right] + \phi_0^k,$$

Dividing both sides by $(1 + \mathrm{E}_0[\Psi_1])$, we obtain (18), where Ψ_1, χ_G, and χ_Ψ are given in (19), and

$$\chi_0^k = \frac{1}{1 + \mathrm{E}_0[\Psi_1]} \left(\phi_0^k + \alpha \frac{1}{|\mathcal{I}|} \mathrm{Cov}_0 \left[G_0 R_1^G + m_1^\top \theta_1 \Psi_1, \left(\alpha \frac{1}{|\mathcal{I}|} \frac{m_{i,1}^\top \theta_1}{m_{i,1}^\top \Omega_{R_2}^{-1} m_{i,1}} + \frac{m_{i,1}^\top \Omega_{R_2}^{-1} m_{i,1}}{m_{i,1}^\top \Omega_{R_2}^{-1} m_{i,1}} \Psi_1 \right) R_1^k \right] \right).$$

We note that the exact expressions for excess returns, (A-23) and (18), include the terms $\mathrm{Cov}_0\left[\Psi_1, \Psi_1 R_1^k\right]$ and $\mathrm{Cov}_0\left[R_1^G, \Psi_1 R_1^k\right]$ that are rather hard to interpret empirically. Using Stein's lemma to obtain the approximations[22]

$$\mathrm{Cov}_0\left[\Psi_1, \Psi_1 R_1^k\right] \approx 2\mathrm{E}_0[\Psi_1]\,\mathrm{Cov}_0\left[\Psi_1, R_1^k\right] \text{ and } \mathrm{Cov}_0\left[R_1^G, \Psi_1 R_1^k\right] \approx \mathrm{E}_0[\Psi_1]\,\mathrm{Cov}_0\left[R_1^G, R_1^k\right],$$

and substituting these terms into (A-22), dividing both sides by $(1 + \mathrm{E}_0[\Psi_1])$, and collecting similar terms on the RHS, we could get rid of the complicated terms, but we would still obtain the form (18) with slightly different χ_G, χ_Ψ, and χ_0^k terms. □

[22] Stein's lemma says that if X and Y are normally distributed, then $\mathrm{Cov}[X, g(Y)] = \mathrm{E}[g'(Y)]\,\mathrm{Cov}[X, Y]$. Because Ψ_1 is not necessarily normally distributed, we only use this as an approximation to obtain testable predictions instead of thinking about this result as a precise statement.

Appendix C Tables

Table 1
Data Summary Statistics

This table reports summary statistics of the bonds (Panel A) and stocks (Panel B) used for six different countries: United States (US), Germany (GE), United Kingdom (UK), Canada (CA), Japan (JP), and Switzerland (SW). Panel A reports the average number of bonds used each day to calculate the term structure (ts) and the illiquidity proxy (illiq). To estimate the term structure, we use bonds of maturities ranging from 3 months to 10 years. To calculate the illiquidity measure, we eliminate bonds of maturities less than one year. Panel B shows country-level summary statistics, monthly mean and volatility, for the stocks used in our sample. The data runs from January 1990 to December 2013.

Panel A: Bonds Summary Statistics

	US		GE		UK		CA		JP		SW	
	ts	illiq	ts	illiq	ts	illiq	ts	illiq	ts	illiq	ts	illiq
1990-2000	124	99	151	130	16	13	44	35	100	92	31	27
2001-2007	77	61	52	42	12	9	20	16	155	133	15	10
2008-2013	146	122	39	32	17	13	27	21	164	138	12	9
All	115	93	105	90	17	13	37	30	127	111	23	19

Panel B: Stocks Summary Statistics

	All	US	GE	UK	CA	JP	SW
Number of stocks considered	11,865	1,815	1,022	3,420	1,927	3,350	331
Average number of traded stocks	6,204	854	529	1,365	932	2,324	201
Average return (ann. in %)	8.12	9.73	7.49	8.51	10.11	0.95	10.63
Volatility (ann. in %)	16.2	15.2	21.4	16.8	19.2	20.8	16.7
Average excess return (ann. in %)	5.17	6.78	4.54	5.56	7.16	-2.00	7.68
Excess return volatility (ann. in %)	16.3	15.2	21.4	16.8	19.2	20.9	16.8

Table 2
Summary Statistics of Illiquidity Proxies

Panel A reports summary statistics (mean, standard deviation, maximum and minimum) for six different country specific illiquidity proxies in basis points. The countries are the United States (US), Germany (GE), United Kingdom (UK), Canada (CA), Japan (JP), and Switzerland (SW). Panel B reports the fraction of being in the low, medium, or high illiquidity bin. Panel C reports the unconditional correlation between the country-specific illiquidity measures. Panel D reports the estimated coefficients with the associated t-statistic and R^2 from the following regression: $\text{Illiq}_t^j = \beta_0^j + \beta_1^j \text{Illiq}_t^G + \epsilon_t^j$, where Illiq_t^j is the illiquidity proxy of country j and Illiq_t^G is the global illiquidity proxy. t-statistics are calculated using Newey and West (1987). Data is monthly and runs from January 1990 to December 2013.

	CA	GE	JP	SW	UK	US
Panel A: Summary Statistics						
mean	4.514	4.341	2.586	7.719	4.094	2.765
stdev	3.792	2.430	1.975	6.741	3.254	1.349
max	16.079	12.581	10.629	41.020	20.188	10.577
min	0.696	0.871	0.523	0.916	0.433	1.068
Panel B: Illiquidity Bins						
low illiq	28.67%	15.03%	65.38%	7.34%	29.72%	53.85%
med illiq	39.51%	41.96%	25.17%	18.18%	46.50%	28.67%
high illiq	31.82%	43.01%	9.44%	74.48%	23.78%	17.48%
Panel C: Cross Correlations						
CA	100.00%					
GE	48.28%	100.00%				
JP	41.57%	53.16%	100.00%			
SW	62.09%	63.53%	45.53%	100.00%		
UK	50.93%	58.35%	29.89%	43.08%	100.00%	
US	34.31%	25.46%	20.77%	40.46%	41.07%	100.00%
Panel D: Loading on Global Illiquidity						
β_0	-0.254	0.896	-0.353	-2.212	-1.026	0.435
t-stat	(-0.24)	(1.35)	(-0.63)	(-1.96)	(-1.78)	(1.12)
β_1	1.491	1.078	0.919	3.106	1.601	0.729
t-stat	(3.64)	(4.49)	(4.43)	(6.90)	(6.50)	(4.98)
Adj R2	0.3171	0.4043	0.4455	0.4367	0.4987	0.6017

Table 3
Global Security Market Line

This table reports OLS regression coefficient of the intercept and slope of the SML on global illiquidity:

$$\begin{aligned} \text{intercept}_t &= a_1 + b_1 \text{Illiq}^G_{t-1} + c_1 X_t + u_{1,t}, \\ \text{slope}_t &= a_2 + b_2 \text{Illiq}^G_{t-1} + c_2 X_t + u_{2,t}, \end{aligned}$$

where X_t captures additional controls such as the global market (market), size (sml), value (hml) and momentum (mom) factors, market volatility (market vol), Hong and Sraer (2016) disagreement proxy (disagreement), and the interaction of the latter with global illiquidity (disagreement x illiq). The intercept and slope are estimated using the Fama and MacBeth (1973) methodology. t-statistics reported in parentheses are adjusted according to Newey and West (1987). Data is monthly and runs from January 1990 to December 2013.

	Panel A: Intercept						Panel B: Slope					
constant	0.014	0.019	0.019	0.018	0.019	0.021	-0.015	-0.019	-0.020	-0.019	-0.014	-0.030
t-stat	(4.17)	(3.67)	(3.64)	(3.40)	(3.16)	(3.51)	(-2.80)	(-2.51)	(-2.46)	(-2.35)	(-1.64)	(-2.69)
illiquidity	0.009	0.010	0.010	0.010	0.011	0.011	-0.019	-0.019	-0.019	-0.020	-0.018	-0.020
t-stat	(2.10)	(2.08)	(2.09)	(2.07)	(2.18)	(2.23)	(-2.69)	(-2.69)	(-2.69)	(-2.66)	(-2.28)	(-2.95)
market		-0.001	-0.001	-0.001	-0.001	-0.001		0.001	0.001	0.001	0.001	0.002
t-stat		(-1.01)	(-1.00)	(-0.96)	(-0.98)	(-1.18)		(0.90)	(0.95)	(1.03)	(0.78)	(1.70)
smb			-0.023	-0.001	-0.005				0.016	0.016	-0.024	
t-stat			(-0.12)	(0.00)	(-0.03)				(0.06)	(0.05)	(-0.07)	
hml			0.030	0.090	0.089				0.107	0.070	0.061	
t-stat			(0.17)	(0.45)	(0.44)				(0.35)	(0.20)	(0.18)	
mom				0.089	0.089					-0.068	-0.070	
t-stat				(0.68)	(0.68)					(-0.35)	(-0.35)	
market vol					0.000						0.000	
t-stat					(-0.29)						(-1.61)	
disagreement						-0.001						-0.025
t-stat						(-0.10)						(-0.98)
disagreement x illiq						0.010						0.043
t-stat						(0.89)						(-2.98)
Adj R2	0.008	0.008	0.001	0.000	-0.003	0.007	0.019	0.017	0.010	0.009	0.012	0.042

Table 4
Illiquidity and Beta Sorted Portfolios

This table reports portfolio returns of illiquidity-to-beta sorted portfolios. At the beginning of each calendar month, we sort stocks in ascending order on the basis of their country prevailing illiquidity and the estimated beta at the end of the previous month. The ranked stocks are then assigned to six different bins: Low/High illiquidity, and low/mid/high beta. Panel A reports our baseline results. Panel B reports results with illiquidity measures have been orthogonalized with respect to the Amihud (2002) market illiquidity measure. Panel C reports results with betas calculated following Karolyi (1992). CAPM Alpha is the intercept in a regression of monthly excess returns on the global market excess return. Returns and alphas are in monthly percent, t-statistics are shown below the coefficient estimates. Beta is the realized loading on the market portfolio. Volatilities and Sharpe ratios are annualized. Data is monthly and starts in January 1990 and ends in December 2013.

	Low Illiquidity			High Illiquidity		
	low β	mid β	high β	low β	mid β	high β
Panel A: Baseline						
Excess Return	0.848	0.616	0.410	0.897	0.803	0.799
t-stat	(3.18)	(1.92)	(0.91)	(3.45)	(2.50)	(2.08)
CAPM alpha	0.794	0.578	0.349	0.854	0.759	0.741
t-stat	(3.02)	(1.84)	(0.79)	(3.23)	(2.32)	(2.07)
Beta	0.649	0.855	1.158	0.658	0.873	1.159
Volatility	15.53	18.63	26.20	15.13	18.72	22.40
Sharpe ratio	0.66	0.40	0.19	0.71	0.51	0.43
Panel B: Illiquidity Orthogonal to Amihud (2002)						
Excess Return	0.837	0.729	0.622	0.917	0.799	0.784
t-stat	(3.34)	(2.04)	(1.38)	(3.95)	(2.41)	(1.83)
CAPM alpha	0.811	0.671	0.540	0.919	0.756	0.709
t-stat	(3.27)	(1.92)	(1.21)	(4.02)	(2.26)	(2.63)
Beta	0.248	0.806	1.414	0.223	0.698	1.285
Volatility	14.51	20.67	26.18	13.43	19.20	24.85
Sharpe ratio	0.69	0.42	0.29	0.82	0.50	0.38
Panel C: Karolyi (1992) Betas						
Excess Return	0.870	0.632	0.399	0.966	0.864	0.716
t-stat	(3.33)	(1.87)	(0.89)	(4.53)	(2.60)	(1.72)
CAPM alpha	0.845	0.591	0.326	0.947	0.814	0.652
t-stat	(3.29)	(1.79)	(0.73)	(4.47)	(2.42)	(1.54)
Beta	0.268	0.776	1.649	0.215	0.724	1.276
Volatility	15.14	19.59	25.99	12.33	19.24	24.14
Sharpe ratio	0.69	0.39	0.18	0.94	0.54	0.36

Table 5
Betting-against-beta Portfolios

This table reports estimated excess returns and alphas of a trading strategy that each month constructs a betting-against-beta strategy in each country and then sorts according to their illiquidity level into two bins (low and high). HML is the high-illiquidity minus the low-illiquidity portfolio. Panel A reports our baseline results. Panel B reports results with illiquidity measures have been orthogonalized with respect to the Amihud (2002) market illiquidity measure. Panel C reports results with betas calculated following Karolyi (1992). Alphas are in monthly percent and t-statistics are adjusted according to Newey and West (1987). Volatilities and Sharpe ratios are annualized. Data runs from January 1990 to December 2013.

	Low Illiq	High Illiq	HML
Panel A: Baseline			
Excess Return	0.598	1.368	0.770
t-stat	(3.30)	(5.23)	(3.00)
CAPM alpha	0.605	1.459	0.854
t-stat	(3.30)	(5.73)	(3.38)
Volatility	10.64	15.35	15.08
Sharpe ratio	0.67	1.07	0.61
Panel B: Illiquidity Orthogonal to Amihud (2002)			
Excess Return	0.643	1.499	0.856
t-stat	(3.17)	(6.30)	(3.52)
CAPM alpha	0.634	1.506	0.872
t-stat	(3.11)	(6.35)	(3.60)
Volatility	11.86	13.92	14.21
Sharpe ratio	0.65	1.29	0.72
Panel C: Karolyi (1992) Betas			
Excess Return	0.931	1.635	0.704
t-stat	(4.22)	(6.02)	(3.48)
CAPM alpha	0.959	1.676	0.717
t-stat	(4.42)	(6.22)	(3.51)
Volatility	12.91	15.87	11.84
Sharpe ratio	0.87	1.24	0.71

Table 6
Market Price of Illiquidity Risk

This table reports estimated factor premia for global market (market), illiquidity (illiquidity), size (sml), value (hml), and momentum (mom) factors using 25 portfolios sorted on global market and illiquidity. Panel A reports standard Fama and MacBeth (1973) estimates. Panel B reports estimates adjusted for omitted factors using the three-pass method of Giglio and Xiu (2017). Data is monthly and runs from January 1990 to December 2013.

	Panel A		Panel B	
constant	1.140	0.114	0.160	0.200
t-stat	(3.80)	(0.33)	(1.23)	(1.54)
market	-0.780	-0.764	0.300	0.327
t-stat	(-1.59)	(-1.19)	(1.36)	(1.47)
illiquidity	-0.426	-0.347	-0.178	-0.173
t-stat	(-3.52)	(-3.08)	(-3.09)	(-3.05)
smb		-0.589		-0.034
t-stat		(-1.05)		(-0.79)
hml		1.153		0.009
t-stat		(1.34)		(0.22)
mom		-0.378		0.107
t-stat		(-0.26)		(1.04)
Adj R2	0.693	0.603	0.426	0.250
3-pass			Yes	Yes

Appendix D Figures

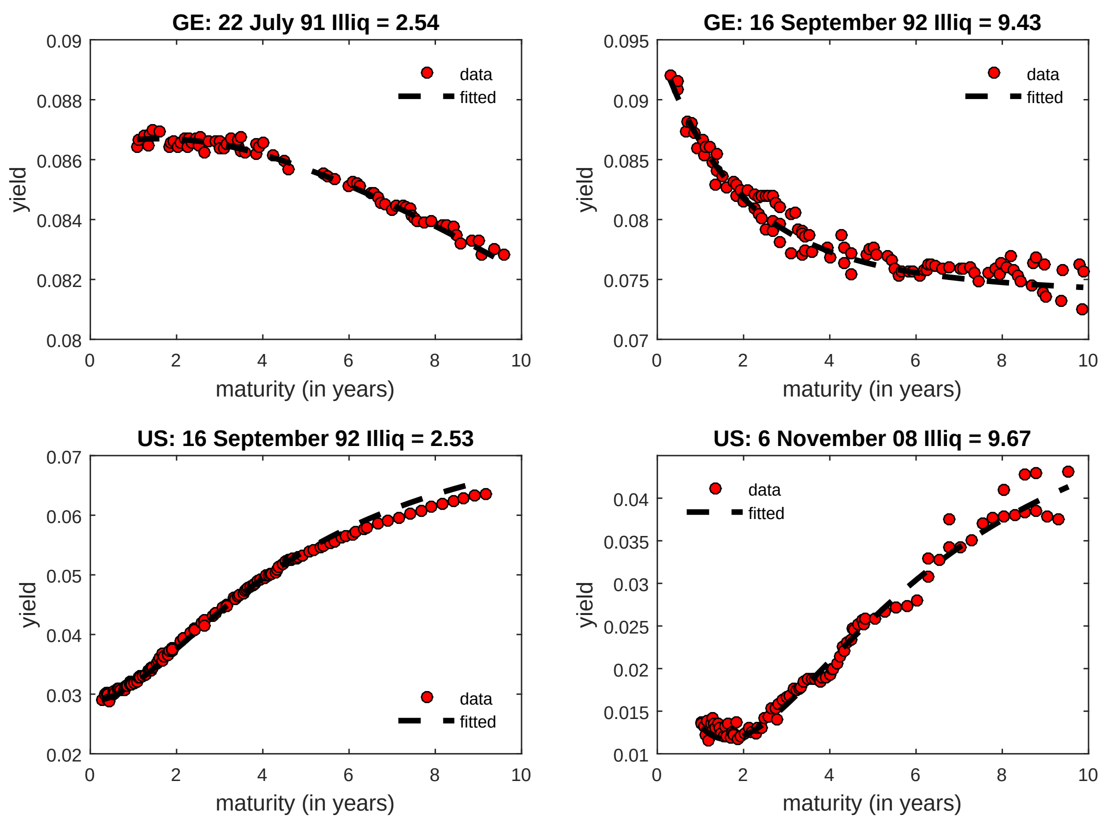

Figure 1. International Term Structures on Different Days

This figure presents data and model-implied yields for Germany and the US for three specific days. The dots are observed yields (in decimals) for different maturities. The dashed line is the fitted curve using the Svensson (1994) method. The upper left panel plots the German term structure on 22 July 1991. The upper right panel plots the German term structure on the day when the British Pound exited the European Exchange Rate Mechanism, 16 September 1992. The lower left panel plots the US term structure on this same day and the lower right panel plots the US term structure on 6 November 2008.

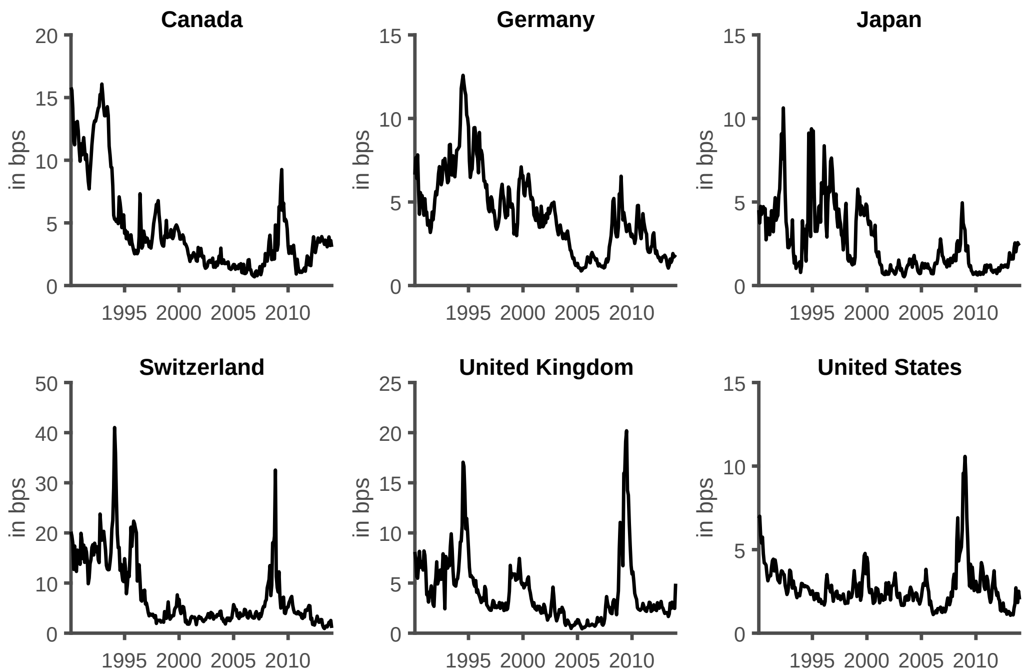

Figure 2. Illiquidity Measures in All Countries

This figure plots country-level illiquidity proxies for six different countries: United States, Germany, United Kingdom, Canada, Japan, and Switzerland. Illiquidity is calculated as the average squared deviation of observed bond prices from those implied by a fitted yield curve using the method of Svensson (1994). Measures are in basis points. Data is monthly and runs from January 1990 to December 2013.

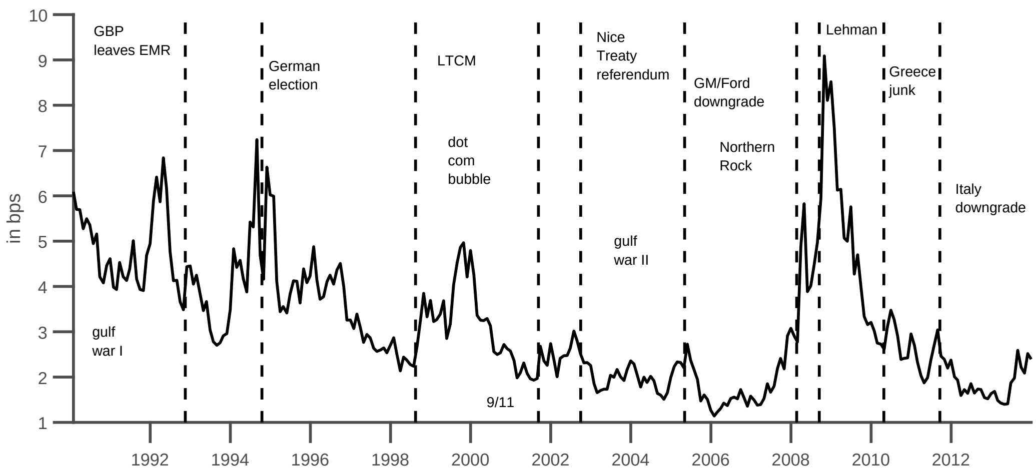

Figure 3. Global Illiquidity

This figure presents global illiquidity in basis points. Global illiquidity is calculated as the market capitalization-weighted average from the six country-specific illiquidity proxies (United States, Germany, United Kingdom, Canada, Japan, and Switzerland). Data is monthly and runs from January 1990 to December 2013.

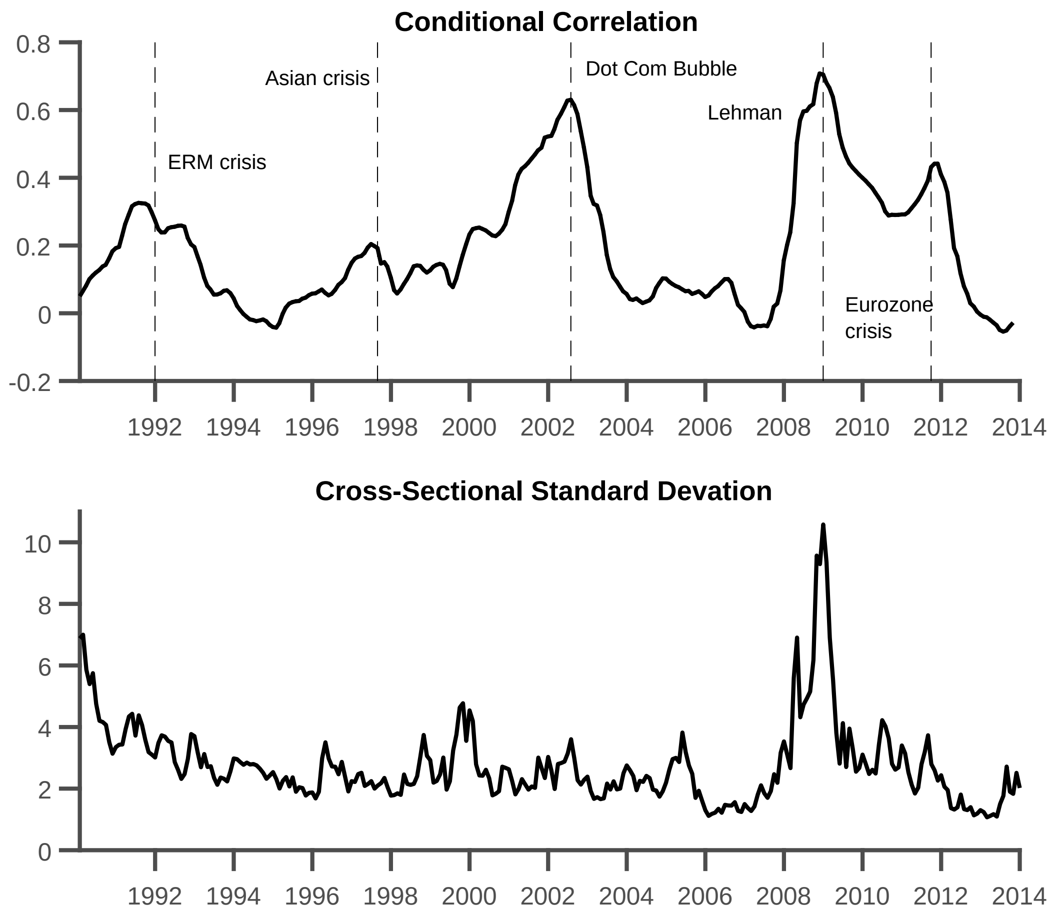

Figure 4. Average Conditional Correlation and Cross-Sectional Standard Deviation

The upper panel of this figure present the conditional average correlation among all six country-specific illiquidity proxies (United States, Germany, United Kingdom, Canada, Japan, and Switzerland). Conditional correlations are calculated using a rolling window of three years using daily data. The lower panel depicts the cross-sectional standard deviation of country-specific illiquidity measures (United States, Germany, United Kingdom, Canada, Japan, and Switzerland). The standard deviations are calculated from monthly data. Data is sampled monthly and runs from January 1990 to December 2013.

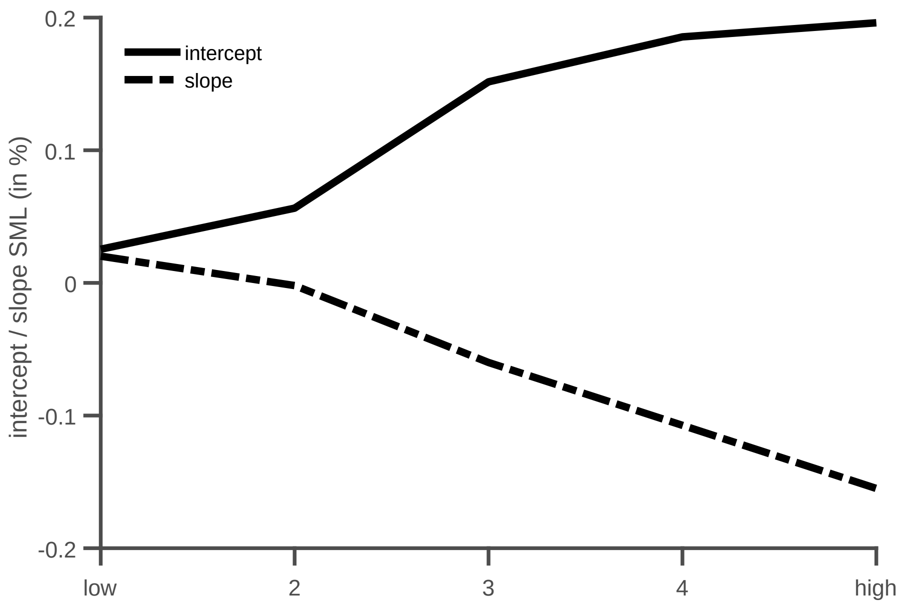

Figure 5. Intercept and Slope of the Security Market Line

This figure plots the average monthly intercept and slope of the security market line for different global illiquidity quintiles. The sample period is from January 1990 to December 2013.